Simplified Approach for Water Distribution Network Dividing
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ABSTRACT
Water Network Partitioning (WNP) is among the most attractive strategies for the improvement of Water Distribution Networks (WDNs) management. The choice of dividing large-scale networks into permanent smaller and manageable subsystems, called District Metered Areas (DMAs), offers precious advantages for the pressure management, the control of consumption, and the monitoring of leaks. On the other hand, WNP may worsen the hydraulic performance and the reliability of the system due to the reduced number of flow paths. In recent years, a number of works were dedicated to the design of DMAs. Anyway, they are mostly based on heuristic multi-objective optimization algorithms, which fail to define wide-ranging principles based on deterministic procedures able to generalize design criteria for an optimal and easy to realize water network partitioning. The aim of this paper is to show how to face the arduous task of the WNP, arranged in clustering and dividing phases, with a novel simplified dividing procedure, based on network hydraulic features, that may be also used as a preliminary procedure to define a starting point for heuristic optimization algorithms, thus reducing the computational burden. First, the clustering layout is obtained by exploiting the properties of the normalized Laplacian matrix L of the un-weighted graph of the network; then, the dividing phase is carried out by means of a novel simplified approach, defined SWND (Simplified Water Network Dividing), based on the ranking of boundary pipes according to some geometric features (i.e. diameter, length, hydraulic conductivity). Ranked pipes are closed one by one in reverse order, stopping when a network performance constraint is not satisfied anymore. The proposed dividing procedure has been compared with a heuristic genetic algorithm based on the minimization of a constrained multi-objective function. The comparison has been made for a real water network, evaluating some hydraulic performance indices, in order to validate the proposed procedure.

Keywords: Water Network Partitioning, Spectral Clustering, District Metered Areas

1 Introduction
In the last years, the “divide and conquer” paradigm has been gaining attention in the study of several complex systems in many research fields. It is based on the community structure principle, according to which a complex system can be easily analyzed if it is subdivided into sub-regions with specific and common characteristics. Finding the communities within a network is a powerful tool for understanding its functioning and development, and for identifying a hierarchy of connections within a complex architecture, especially in the case of complex, huge and dynamic systems [1].
In this regard, recently Water Network Partitioning (WNP) [2] has been gaining attention in the management of Water Distribution Networks (WDNs), becoming among the most attractive strategies for network control improvement. The option of dividing large-scale networks into smaller and manageable subsystems, called district metered areas (DMAs), offers undisputable advantages for the monitoring and control of leakage [3], for the pressure management [4], for the water quality monitoring [5], and for speeding up the repairing interventions [6]. Finally, combining the water network partitioning and innovative Information and Communications Technology (ICT), remote-controlled devices and big data analysis, it is possible to change the traditional WDN management approach, transforming water networks into modern Smart Water Network (SWAN), simplifying the analysis of data and information about the system, collecting them for each DMA. On the other hand, WNP may worsen the hydraulic performance and the robustness of the system due to the reduced number of available flow paths.

In the scientific literature, a number of works were dedicated to the design of DMAs. Most of them are based on the application of decomposition algorithms using graph and spectral theories (e.g., [7]; [8]; [9]; [10]). Recent works use the concept of modularity function for DMA design, to account for the very close nodal interconnections existing between some node groups of WDN layout ([11]; [12]; [13]). According to [12], a valid procedure to obtain an optimal WNP is carried out in two main phases: a) clustering, in which the optimal number, shape and size of the districts are defined by balancing the number of nodes of each cluster and simultaneously by minimizing the number of edge-cuts $N_{ec}$ ([14]; [15]; [16]; [17]), and b) dividing, in which the proper position of the gate valves and flow meters along the boundary pipes is defined by preserving the hydraulic performance and the continuity of the network and minimizing the economic investment ([18]; [19]). The number of possible solutions in terms of gate valves/flow meters positioning grows enormously with network size. Generally, a heuristic procedure, defined here Heuristic Water Network Dividing , HWND, is required in this phase [20], because the choice of the optimal positioning of flow meters and gate valves is a NP-hard problem [21]. Anyway, by designing DMAs with a heuristic approach, traditional methods fail to define wide-ranging principles based on deterministic procedures able to generalize design criteria for an optimal and easy to realize water network partitioning. Starting from this point, the aim of this paper is to introduce a novel simplified approach (Simplified Water Network Dividing, SWND) to face the arduous task of the WNP. Based only on geometric aspects, it can be also used as a preliminary procedure to define a starting point for heuristic optimization algorithms, reducing the computational burden, and providing some useful general insights for the water network partitioning.

The clustering layout is obtained with a Spectral Clustering (SC) algorithm, which exploits the properties of the normalized Laplacian matrix $L_{rw}$ of the un-weighted graph of the network [22]. The elements of the system are clustered through the corresponding coordinates of the eigenvectors of $L_{rw}$, with a fast and elegant analytical procedure.

Regarding the dividing phase, it is well-known that its aim is to choose whether either a gate valve must be closed or a flow meter must be installed along any boundary pipe. The main novelty of the present work regards this phase. In fact, network dividing is not carried out through a heuristic optimization algorithm, but with a simple procedure, which can be easily adopted by practitioners in charge of WDN management. The boundary pipes are ordered according to some geometric and hydraulic characteristics (diameter, length, hydraulic conductivity), and then they are closed one by one in reverse order, checking every time that the chosen performance constraint is satisfied (i.e. the minimum pressure in the network does not fall below a desired threshold value $h_0$). The procedure stops when the closure of another pipe would lead to undesired performance conditions. In this way, it is possible to simplify the design of an optimal WNP and simultaneously detect its negative impact on the WDN performance. The proposed procedure, that required only few hydraulic simulations, has been compared with a heuristic genetic algorithm based on the minimization of a
constrained multi-objective function, which requires thousands of hydraulic simulations of the network, exploring the space of the possible choices of gate valves and pipe closures to search for the optimal configuration. The comparison has been made for a real water network, serving the city of Parete (close to Naples), by evaluating some hydraulic performance indices for different DMA layout. The aim is to provide a simple tool for WNP, which can be also adopted for setting the initial guess of heuristic optimization algorithms, improving their results with reduced computational burden.

2 Methods

According to [12], the methodology for network partitioning proposed in this work consists of two phases, clustering and dividing, both described hereinafter.

Water distribution networks can be modelled as an un-weighted [23] graph \( G = (V, E) \), where \( V \) is the set of \( n \) vertices \( v_i \) (or nodes) and \( E \) is the set of \( m \) edges \( e_l \) (or links). Starting from this assumption, it is possible to define some basic graph matrices that describe WDN topology. In particular, the symmetric adjacency \( nnm \) matrix \( A \) expresses the connectivity of the graph, where elements \( a_{ij}=a_{ji}=1 \) indicate that there is a link between nodes \( i \) and node \( j \) and \( a_{ij}=a_{ji}=0 \) otherwise. The degree of a node is the number of its connections to other nodes, \( k_i \). The diagonal degree matrix \( D_n \) contains the degree of all nodes. The laplacian matrix of the graph, \( L \), is defined as \( D = D_n - A \). The graph schematization of WDNs and the definition of the adjacency, degree of a node is defined as the number of its connection

\[
N_{DL} = \binom{N_{ec}}{N_{fm}}
\]

Since \( N_{DL} \) becomes huge number already for relatively small WDNs, it is often impossible to investigate all the solution space. Therefore, the use of heuristic optimization techniques is necessary to find the optimal positioning of devices along the boundary pipes. Contrary to this approach, in this paper a simplified procedure for the dividing phase is proposed. The novel procedure aims to minimize the number of flow meters ensuring a suitable pressure service level for the users. Specifically, the simplified procedure SWND consists of the following steps:

1) defining a performance constraint (i.e. minimum service level, minimum resilience, etc.);
2) ranking the boundary pipes according to a chosen hydraulic/geometric characteristic (e.g. diameter \( D \), length \( L \), resistance coefficient \( L^{0.5}/D^{2.5} \)).
3) closing the j-th pipe with the highest/lowest chosen hydraulic/geometric characteristic;
4) carrying out the hydraulic simulation of the WDN;
5) checking the performance constraint (if it is satisfied go to step 3, otherwise go to step 6);
6) locate gate valves along closed pipes and flow meters along the other boundary pipes.

Steps 2 and 5 represent the novel way to carry out the dividing phase. The boundary pipes are sorted according to their geometric or hydraulic features and then closed one by one until the hydraulic constraint (step 5) is fulfilled. The SWND is based on the simple idea to close boundary pipes with the smallest diameter or the smallest conductivity, as it is reasonable to expect that they carry small water flows. It consists of few steps – the proper number of closed gate valves \( N_{gv} \) and installed flow meters \( N_{fm} \) are defined by carrying out only \( N_{gv}+1 \) hydraulic simulations – very easy to apply without carrying out thousands of simulations, as with heuristic optimization algorithms.

For the case of a real WDN, the proposed dividing procedure was compared to the solutions provided by the Heuristic Water Network Dividing (HWND) achieved with Non-dominated Sorting Genetic Algorithm II (NSGA-II) [24], that was run considering a number of decisional variables equal to the number of boundary pipes. Each decisional variable can assume two possible values, that are 0 and 1, corresponding to the closure of the boundary pipe with a gate valve, and to the installation of a flow meter, respectively. The objective functions (OF) to be assessed for each iteration are:

\[
OF1 = \min\left(N_{fm}\right)
\]

\[
OF2 = \max\left(d^3H\right)
\]

where \( N_{fm} \) is the number of flow meters, \( d^3 \) and \( H \) are the vector of users’ demands and of total heads at demanding nodes, respectively. To ensure an adequate level of service for the users in terms of node pressure, the minimization of \( OF1 \) and the maximization of \( OF2 \) were achieved with the constraint of Eq. 3:

\[
\text{constraint: } h_i > h^*_i
\]

in which, \( h_i \) and \( h^*_i \) are the actual and the design pressure heads at the i-th node of the network. The NSGA-II algorithm was run with a population of 350 individuals and for 100 generations with a crossover percentage \( P_{\text{cross}}=80\% \) and a mutation rate \( P_{\text{mut}}=2\% \).

In order to validate the procedure, different partitioning layouts have been designed with a different number of districts (from 2 to 6 DMAs). Finally, the effectiveness of the proposed methodology has been tested using several performance indices: minimum \( h_{\text{min}} \), mean \( h_{\text{mean}} \), and maximum \( h_{\text{max}} \) node pressure head, resilience index \( I_r \) [25], deviation of resilience index \( I_{rd} \) [18] and energy efficiency index \( e \) [26].

3 Case Study and Results

The proposed procedure was tested on the WDN of Parete, a small town located in a densely populated area south of Caserta (Italy), with a population of 11,150 inhabitants. The hydraulic model of the WDN consists of 182 demanding nodes, 282 pipes and 2 sources with fixed head of 110 m a.s.l. A uniform design pressure \( h^*_i=19 \) m was assumed. The hydraulic simulations were carried out for one day of WDN operation, using the software EPANET [27]. Reference was made to the day of maximum consumption in the year when the total demand from the nodes in the morning and midday peaks is 77.2 L/s. Table 1 compares the results of the partitioned layouts (from 2 to 6 DMAs), obtained all with the same clustering phase with Spectral Clustering (SC) algorithm and with the simplified procedure SWND, carried out with two different choices for pipe ranking.
(the inverse of the pipe diameter 1/D and a resistance coefficient L^{0.5}/D^{2.5}), and with the HWND procedure. The number of boundary pipes N_{bc}, of flow meters N_{fm}, of gate valves N_{gv}, and the hydraulic performance indices are reported, compared with the performance of the Original Water Network (OWN). Table 1 also reports the number of all possible dividing combination N_{DL} for each dividing procedure, highlighting the computational complexity of the water network partition problem. HWND with NSGA-II indicates the results of the heuristic dividing obtained setting the same number of gate valves and flow meters as provided by the simplified SWND procedure, as explained below.

Table 1. Results for WDN partitioning of Parete from 2 to 6 DMAs. (*Results of NSGAII with the same number of gate valves provided by SWNP*)

<table>
<thead>
<tr>
<th>Network Layout</th>
<th>Clustering phase</th>
<th>Dividing phase</th>
<th>N_{DL}</th>
<th>N_{fm}</th>
<th>N_{gv}</th>
<th>h_{min}</th>
<th>h_{mean}</th>
<th>h_{max}</th>
<th>I_{r}</th>
<th>I_{rd}</th>
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<tr>
<td>OWN</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>21.36</td>
<td>31.05</td>
<td>50.47</td>
<td>0.481</td>
<td>-</td>
<td>0.859</td>
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<td>WNP 2 DMA</td>
<td>SC</td>
<td>SWND (1/D)</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>21.20</td>
<td>29.22</td>
<td>49.74</td>
<td>0.401</td>
<td>0.166</td>
<td>0.836</td>
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<td></td>
<td></td>
<td>SWND (L^{0.5}/D^{2.5})</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>21.20</td>
<td>29.22</td>
<td>49.74</td>
<td>0.401</td>
<td>0.166</td>
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<td></td>
<td></td>
<td>HWND (NSGA-II)</td>
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<tr>
<td>WNP 3 DMA</td>
<td>SC</td>
<td>SWND (1/D)</td>
<td>9</td>
<td>1</td>
<td>9</td>
<td>19.45</td>
<td>29.41</td>
<td>50.16</td>
<td>0.423</td>
<td>0.121</td>
<td>0.844</td>
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<td></td>
<td></td>
<td>SWND (L^{0.5}/D^{2.5})</td>
<td>9</td>
<td>1</td>
<td>9</td>
<td>19.99</td>
<td>29.41</td>
<td>50.16</td>
<td>0.423</td>
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<tr>
<td>WNP 4 DMA</td>
<td>SC</td>
<td>SWND (1/D)</td>
<td>16</td>
<td>6</td>
<td>10</td>
<td>21.63</td>
<td>29.64</td>
<td>50.41</td>
<td>0.424</td>
<td>0.119</td>
<td>0.844</td>
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<td></td>
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<tr>
<td>WNP 5 DMA</td>
<td>SC</td>
<td>SWND (1/D)</td>
<td>20</td>
<td>8</td>
<td>12</td>
<td>22.75</td>
<td>30.50</td>
<td>50.02</td>
<td>0.425</td>
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<td>8</td>
<td>12</td>
<td>22.75</td>
<td>30.50</td>
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<td>HWND (NSGA-II)</td>
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<tr>
<td>WNP 6 DMA</td>
<td>SC</td>
<td>SWND (1/D)</td>
<td>23</td>
<td>11</td>
<td>12</td>
<td>21.05</td>
<td>29.92</td>
<td>50.49</td>
<td>0.434</td>
<td>0.098</td>
<td>0.846</td>
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<td></td>
<td></td>
<td>SWND (L^{0.5}/D^{2.5})</td>
<td>23</td>
<td>11</td>
<td>12</td>
<td>21.05</td>
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SWND and HWND provided identical solutions for 2 and 3 DMAs, as the number of possible combinations N_{DL} are in these cases very small: N_{DL}=4 with N_{fm}=1 for 2 DMAs and N_{DL}=9 with N_{fm}=1 for 3 DMAs. Therefore, the use of optimization algorithms results unnecessary and time consuming in this case and it is advisable to apply simplified procedures like the SWND.

As the number of districts increases (and the number of all possible device positions dramatically increases, as reported in Table 1), the two procedures provide different solutions in terms of minimum number of flow meters; for example, for 4 DMAs, N_{fm}=4 for the HWND, N_{fm}=6 for the SWND with 1/D and N_{fm}=7 for the SWND with L^{0.5}/D^{2.5}. It is worth to highlight that the differences of number of flow meters range between 2 and 3 in all analyzed cases. This issue may cause a slight increase of partitioning costs [17]. Furthermore, a small number of flow meters simplifies the water balance computation for WDNs.
From the hydraulic performance point of view, all the solutions proposed in Table 1 satisfy the required service level for the users. Indeed, the minimum value of nodal pressure is always greater than the design pressure and, in some cases, \( h_{\text{min}} \) even exceeds the value of the original (non-partitioned) water network. Furthermore, the mean and the maximum values of nodal pressure are almost equivalent. The resilience index deviation is always smaller than 22\%, with a minimum value of 5\% corresponding to the solutions with 4 and 5 DMAs obtained with HWND with NSGA-II*. The alteration of energy efficiency index is always smaller than 2\% for all the partitioning layouts. In order to further validate the simplified approach SWND, the partitioning layouts with 4, 5 and 6 DMAs (solutions with 2 and 3 DMAs are identical) were analyzed with HWND setting the same number of flow meters provided by the SWNP, as reported in Table 1 at the fourth rows of each number of DMAs (solution layouts indicated as HWND with NSGA-II*). It is clear that the hydraulic performance of the partitioned layouts provided by HWND is in all cases just slightly better than that of SWND, validating the simplified approach as preliminary tool for the water network partitioning task. The partitioning layouts for 6 DMAs obtained with SWND with \( L^{0.5}/d^{2.5} \) and with HWND with NSGA-II* are shown in Figure 1.

It is interesting to note that, most part of the hydraulic device (flow meters and gate valves) are located in the same boundary pipes for the SWND and the HWND, highlighting that, it is possible to define useful design insights for the water network partitioning (Figure 1). For example, all the partitioning layouts show that, on the pipe set with the lowest diameter, gate valves are always installed. Therefore, this aspect, which requires further investigations, could be exploited to reduce the possible dividing combinations \( N_{DL} \), setting a priori gate valves on the boundary pipes with the lowest diameter. In this regard, it could be also possible to improve the performance of heuristic procedures by applying the partitioning layout evaluated by SWND as the initial guess of the optimization problems. It is also interesting to note, for example in the case of 6 DMAs, that the number of all possible dividing layout, considering \( N_{fm}=9 \), is \( N_{DL}=817190 \). Thus, also the HWND procedure, that has analyzed just the 4\% of all them, likely provides a sub-optimal partitioning solution. The solution given by the simplified procedure SWND, that represent just 1 of the 817190 possible combinations, constitutes itself a valid sub-optimal solution that satisfies the hydraulic
performance without requiring any heuristic approach. It can be considered a valid preliminary solution to the arduous problem of water network partitioning.

4 Conclusions

In this paper a simplified approach, defined SWND, for water network dividing was proposed and compared with an alternative procedure based on the use of the heuristic optimization algorithm HWND. The comparison was developed for the case of a real medium-sized WDN. The two procedures provide the same results for small number of DMAs. However, also with increasing the number of DMAs, the results are only slightly different. The heuristic HWND procedure provided solutions that satisfy the hydraulic performance with a little smaller number of flow meters, compared to SWND; but the values of the hydraulic performance indices result comparable, showing that SWND is able to provide a suitable sub-optimal partitioning layout. Hence, the proposed simplified procedure represents a good alternative to more complex optimization procedures, which cannot be easily adopted by practitioners in charge of WDN management, as they require expert skill and powerful computational tools. It is important to highlight that, the simplified dividing phase is suitable thanks to the effectiveness of the used spectral clustering algorithm, that is able to minimize the boundary pipe set, ensuring a less performance deterioration in the subsequent dividing phase. Alternatively, the partitioning layouts provided by SWND could be adopted as initial guess for HWND procedure.

Further works will test the simplified approach on larger water distribution networks with a greater number of boundary pipes, stressing the methods in order to understand if it is possible to face the arduous problem of the water network partitioning also without heuristic tools. The aim is also to generalize useful insights about the reduction of the solution space, for example through the a priori closure of part of the boundary pipe set.

5 References


