Heat Exchanger Network Optimization and Controllability Using Design Reliability Theory

Rodolfo Tellez and Brent Young*

*Department of Chemical and Petroleum Engineering,
University of Calgary, 2500 University Drive NW, Calgary, Alberta T2N 1N4*

**E-mail: byoung@ucalgary.ca*

Abstract

For a given heat exchanger network (HEN) it is often necessary to determine its behaviour to disturbances in supply temperature and/or inlet flow rate variations, that is its ability/flexibility to meet the design requirements at new operating conditions. An analysis of the HEN flexibility is very useful to assess other design options and in the design of a robust control structure. The use of design reliability theory coupled with fuzzy design uncertainties can be used to determine the possibility of violating the HEN constraints. This measures the potential of the design failure rather than the frequency of failure, the latter being measured by probability theory.

The HEN constraints considered in this work are the target temperatures, hot/cold utility flow rate and heat transfer area/overall coefficient (UA). If the design under consideration results in a significant value of the failure possibility for any of these constraints, then this HEN design will require either a modification of the design or the establishment of a specific requirement in the control system. The case study treated in this paper shows that design reliability theory is a useful tool for determining HEN constraint violations that will require special attention from a control point of view, that is controllability analysis. Thus this approach has proven to be a useful tool for determining design changes and for developing a workable control scheme for HEN designs.

1. Introduction

A heat exchanger network (HEN) is an arrangement of heat exchangers; in which cold and hot process streams and hot and cold utility streams interchange energy. The purpose of the HEN is to recover energy from the hot process streams to heat up cold process streams using the least amount of

hot and cold utility streams, while achieving specified outlet target temperatures of the process streams. Normally the classical synthesis of a HEN assumes and keeps all design operating conditions at a constant value. Before further design for detailed engineering some analysis and modifications of the synthesised HEN have to be carried out, in order to guarantee good flexibility of the HEN, since in reality operating conditions are expected to change. This paper shows a methodology to optimize a HEN using "Fuzzy Design Reliability Theory" (e.g. Kubic and Stein, [1]). The main advantage in using this methodology is that it incorporates the experience of the design engineer or operations personnel, in order to determine the expected variations in the operating conditions that potentially could happen. These variations will be considered to be the design uncertainties. This new approach to optimization will determine the HEN behaviour in order to assess the detailed engineering of the HEN and the determination of suitable control systems.

Several authors have presented different approaches in the attempt to synthesise an optimum HEN. Türkay and Grossman [2] proposed an algorithm that requires a great deal of computational effort, starting from a large and complex superstructure. Through the use of an advanced robust optimisation technique they synthesised HEN's until an optimum solution was found. Other authors have worked on ways to determine the flexibility of a HEN. Swaney and Grossmann [3] presented a "Flexibility Index". Bansal et al. [4] performed a flexibility analysis using stochastically uncertain parameters. Coldberg et al. [5] presented an approach with two resilient targets for HEN synthesis involving large uncertainties in supply temperatures and heat capacity flow rates. Kotjabasakis and Linhoff [6] used a simpler approach, analysing the paths in which a disturbance propagates through a HEN. They proposed a way to determine the

modifications needed, costing all design alternatives, in order to withstand variations in the operating conditions. Aguilera and Marchetti [7] presented a procedure for the optimization of a HEN on top of the base level control using LP or NLP techniques.

This paper presents another approach to optimize and determine the controllability of a HEN. Following the normal HEN synthesis using the usual pinch analysis design procedures, this proposed approach uses design reliability theory (e.g. Kubic and Stein, [1]) coupled with fuzzy set theory principles (e.g. Ross et al., [8]).

2. Operating conditions variations and membership functions

Engineering design is subject to non-random uncertainties and it is important to verify if a design will work in spite of such uncertainties. Fuzzy set theory is used to measure non-random processes and can be used to quantify the confidence of a process design that is subject to fuzzy uncertainties. After the synthesis of a HEN is completed, it is necessary to determine if such a design can accommodate any variations in the operating conditions that can occur during normal operation. For a HEN, variations in the supply temperatures and heat capacity flow rates are the most significant variations with a high potential of occurrence. Such variations are often difficult to foresee, but on most occasions the knowledge and experience of the design engineers and input from operations people can be used to determine the expected possibility of occurrence. The potential of such variations to occur can be related to a membership function (MF). Zadeh [9] first proposed this cognitive concept. The shape of the MF depends on the way a design engineer or operations engineer conceptualises the expected changes in the operating conditions.

An example of a MF is shown in Figure 1. The truncated triangle has in the middle of the plateau of maximum membership value, or MF core, the nominal operating conditions. The width of the MF core contains all the operating conditions that have the highest possibility of happening. Operating conditions with a low possibility of happening are contained in the lower and upper boundaries of the MF. The base of the MF will represent all possible values of the normal operating condition.

3. Design reliability and unreliability

The models required for the optimum design of any chemical process are expressed as a set of j inequality constraints (g_j).

$$0 \geq g_j(\mathbf{d}, \mathbf{u}, \mathbf{p}) \quad (1)$$

Where $\mathbf{d}$ is the vector of equipment design parameters, $\mathbf{u}$ is the vector of operating conditions and $\mathbf{p}$ is the vector of i fuzzy design parameters. For a given set of parameters, there is a set of operating conditions that violates each constraint. The set of operating conditions for which the process fails F_F is defined in equation 2.

$$F_F = \bigcup_j F_j = \left\{ \mathbf{u} / \max_j g_j(\mathbf{d}, \mathbf{u}, \mathbf{p}) > 0 \right\} \quad (2)$$

This equation (2) implies if one or more constraints are violated for a given $\mathbf{u}$, the vector of operating conditions is infeasible. The design is a failure if the complement of F_F is the empty set. In this case fuzzy set theory is employed because it measures the potential for an event, not the frequency of an event as probability does. Moreover, fuzzy sets can be used to describe the potential realization of an uncertain parameter for which distinct bounds are not known. Fuzzy reliability and unreliability are expressed as:

$$R = \sup_{\mathbf{p}} \min_i [\mu_{p_i}(p_i)] \quad (3)$$

$$\text{subject to } \alpha(\mathbf{d}, \mathbf{p}) \leq 0$$

$$U = \sup_{\mathbf{p}} \min_i [\mu_{p_i}(p_i)] \quad (4)$$

$$\text{subject to } \alpha(\mathbf{d}, \mathbf{p}) > 0$$

where $\alpha(\mathbf{d}, \mathbf{p})$ is the maximum lower bound on g_j with respect to the operating conditions.

$$\alpha(\mathbf{d}, \mathbf{p}) = \inf_w \max_j g_j(\mathbf{d}, \mathbf{u}, \mathbf{p}) \quad (5)$$

If $\alpha > 0$, $\max g_j$ must be positive for all $\mathbf{u}$, and no feasible region exists. However, if $\alpha \leq 0$, at least one set of operating conditions exists that satisfies all constraints, and the result is that the complement of F_F is not an empty set. Fuzzy design reliability measures the possibility that there is some realization of the parameters in the fuzzy set of parameters for which the design works. A design is considered good when reliability is maximised and unreliability is minimised. Since the design operating conditions happen at maximum possibility values, the reliability of the plant will be 1. Thus, it is more convenient to determine the design unreliability and find a minimum value. In order to simplify the optimization problem to a manageable form, the case of fixed operating

conditions will be considered. This simplification requires the solution of equations 6 and 7 at fixed operating conditions, $\mathbf{u}$.

$$U^*(\mathbf{u}) = \max_j \left\{ \sup_p \min_i [\mu_{p_i}(\mathbf{p}_i)] \right\} \quad (6)$$

$$\text{subject to } g_j(\mathbf{d}, \mathbf{u}, \mathbf{p}) > 0 \quad (7)$$

Equality constraints are used to eliminate degrees of freedom in order to reduce the dimensionality of the problem. Hence, the inequality constraints must be satisfied for feasible operating conditions. The following is a modification of the algorithm proposed by Kubic and Stein [1] for the estimation of the design unreliability of HENs:

1. Obtain the initial estimates of the parameters p_i . In this case these will be the flow rates of the bypass streams.

2. Determine the operating conditions by solving the following equation:

$$\alpha(\mathbf{d}, \mathbf{p}) = \inf_u \max_j \left[\frac{g_j(\mathbf{d}, \mathbf{u}, \mathbf{p})}{s_j} \right] \quad (8)$$

Where s_j are the constraint scalings calculated using Equation 9:

$$s_j = \max_i \left| (p_{imax} - p_{imin}) \frac{\partial}{\partial p_i} g_i \right| \quad (9)$$

3. Check convergence by comparing the current set of operating conditions to the set from the previous iteration. If this is the first iteration, this step must be skipped. If convergence is achieved, then halt.

4. Since reliability is more difficult to calculate than unreliability, estimate the unreliability of the HEN using Equation 6.

5. Successive substitution: Set the vector of parameters equal to the parameters determined in step 4 and return to step 2.

For highly non-linear and complex systems this is a good approach to determine the flexibility of a HEN. The Appendix shows a simple example of how these calculations are performed. However, this algorithm does not guarantee either a unique solution or the upper bound value of unreliability (see Appendix). It is also important to note that the sum of reliability and unreliability is not necessarily one, as we are using possibility theory rather than probability theory. Based on possibility theory, for two contradictory events, if one event is highly possible, it will not prevent the contrary event from being highly possible. In contrast with probability theory and its theorems, the probability of an event completely determines the probability of the contrary event. If both fuzzy reliability and unreliability are large, the design may work but also

it may not be able to compensate for all of the uncertainty, implying that the design is not flexible enough.

4. Case study

HEAT EXCHANGER NETWORK

The following case study is based on the HEN analysed by Kontjabasakis and Linhoff [6]. This HEN is comprised of two hot streams (H1 and H2) and two cold streams (C3 and C4), four process-to-process heat exchangers (HX1, HX2, HX3 and HX4), and three process-to-utility heat exchangers (CH1, CH2 and HC4). Design supply temperatures, heat capacity flow rates and target temperatures are shown in Table 1. The HEN schematic is presented in Figure 2.

The HEN was simulated using the HYSYS commercial process simulator. Since a rigorous simulation was used to simulate the HEN, the heat capacity changed with temperature and pressure. The values of WCp were taken at pinch temperatures of 160°C for hot streams and 140°C for cold streams. With these values of WCp , values of the heat transfer area A were determined as shown in Table 2. Therefore, these values necessarily disagree with those obtained by Kontjabasakis and Linhoff, as they simplified the problem by using a set of linear algebraic equations to model the entire HEN. In this case study the values of A were used for the equipment design parameters vector, $\mathbf{d}$. These A values were kept constant throughout all calculations. The size of all heat exchangers and values of the overall heat transfer coefficients were determined using HTFS-TASC commercial software.

FUZZY PARAMETERS AND DESIGN CONSTRAINTS

Fuzzy design parameters are shown in Table 3. Table 4 shows equality and inequality design constraints. If the target temperatures of H1, H2 and C4 are chosen as equality constraints, which implies that the availability of both hot and cold utilities flows should be enough to meet such constraints. In practice, in order to avoid any effect downstream from the HEN we want to keep those temperatures as close to nominal values as possible. However, we have to consider that the objective of the HEN synthesis is to specify the arrangement of heat exchangers with a minimum of operating and capital costs. For this set of target temperatures, the values of g_3 , g_4 and g_5 are 25% above the minimum utility requirements, which implies an availability of 25% excess in utilities. The C3 target

temperature is reached through the use of a process-to-process heat exchanger and the intention of the design is to allow variations of $\pm 5^\circ\text{C}$ from the nominal value of 180°C .

In order to determine the operating conditions, operating degrees of freedom have to be identified or added to the HEN. In this case, four degrees of freedom are added, bypasses on the four process-to-process heat exchangers. By doing this we can change the flow rate through the bypasses in order to meet all design constraints. These four operating degrees of freedom will become manipulated variables to be used to control the HEN. Figure 3 shows all the design variables u , p and d for the HEN. Furthermore, u will be considered as the manipulated variables to control the HEN.

Figures 4 and 5 show the membership functions for the selected operating conditions. In Figure 4, the cores of all the MF's have a width of 10°C , and the MF supports have a width of 30°C . In Figure 5, the cores of all the MF's have a width of 20% of the nominal operating levels and the supports have a width of 50% of the same nominal values. In this case, no composition changes are expected in the hot and cold streams, therefore variations in WCp will be due to flow rate changes only.

UNRELIABILITY RESULTS

Table 5 shows values of "Unreliability" for some operating condition variation scenarios. In some cases, with a full variation of either T_s or WCp , no constraint is violated, and hence, the unreliability will be zero. In some cases, with variations of around minus 15% in WCp_{H1} , the design unreliability is greater than 50% (70.2% in this case) and therefore it fails. For relatively small positive variations in WCp_{H1} and WCp_{H2} , within the range of 100% possibility, the design fails and its unreliability is 100%. Typically, the original design of a HEN cannot be changed without capital expenditures and for changes in operating conditions that would result in an increase in the operating costs. None-the-less it is important to determine the possibility of failure of the original design and suggest the necessary changes. According to Kubic and Stein [1], an unreliability value of less than 50% would be considered a good value for the design. Without considering the costs of a change in the hot and cold utilities, if the available amount of both hot utility for HC4 and cold utility for CH2 is +50% of the minimum amount required, then the HEN unreliability changes, as observed in Table 6. Although the overall unreliability of the design is still a large value of 96.7% with the violation of g_2 , some

constraints are no longer violated for either large or small variations in the operating conditions.

If the same hot and cold utilities are increased to 75% of the minimum amount required, the "Unreliability" of the design is shown in Table 7. The overall unreliability is 96.7% with the violation of g_2 . We can continue to increase the availability of hot and cold utilities and eventually the only two design constraints that will be violated will be g_1 and g_2 . This is warning us that the control system for the HEN has a limitation, in that it is not keeping the C3 target temperature within an acceptable range of $\pm 5^\circ\text{C}$ at 180°C for some extreme variations in the operating conditions. Another alternative to optimize the HEN operation is to implement certain control strategies upstream of the HEN, keeping the variations in WCp within a narrow range. The corresponding MF's will also be modified. This could be achieved using surge tanks or with tight flow controllers. Of course, this kind of modification will be subject to a cost evaluation and compared to the cost of increasing the hot and cold utilities availability. For example, one option could be that the cores of all WCp MF's are shrunk to a width of 10% of the nominal values and the bases are also shrunk to 30% of the same nominal values. Figure 6 shows the new MF's and Table 8 shows the values of the "Unreliability". In this case the overall "Unreliability" of the HEN design was 4.3%, and not 96.7%, achieving the desired under 50% "Unreliability".

DYNAMIC STUDY

Single loop controllers were implemented on CH1, CH2 and HC4, and tuned using the Ziegler-Nichols open loop tuning technique. No control was implemented for the target temperature of C3, T_{T-C3} . To observe the dynamic behaviour of the design, step variations in the operating conditions were applied; Figure 8 shows one example of how the step variations in the operating conditions are made with respect to the membership functions. White noise is added to the operating conditions to provide realism to the simulation. Figures 9 and 10 show the dynamic response of the target temperatures and utility consumption to variations in the supply temperatures. Figures 11 and 12 show the dynamic response of the HEN to variations in the inlet flow rates to the HEN. In both cases the demand for steam for HC4 reaches the maximum available, but the target temperature of C4 remains within an acceptable range of $\pm 5^\circ\text{C}$ from the desired setpoint.

Since T_{T-C3} can be modified by manipulating four degrees of freedom, this is a good candidate for a

MIMO controller, such as a model predictive controller (MPC). Figure 13 shows the implementation of a 4x1 MPC controller, while Figures 14 and 15 show the dynamic response of the HEN to supply temperature variations. Figures 16 and 17 show the dynamic response to changes in the input flow rates. The MPC controller shows good performance in moving the target temperature of C3 to the specified setpoint. Although this application of an MPC is sluggish compared to the single loop controller, it is more robust and will always move the controlled variable to its set point. Additionally, if the percentage openings of the bypass valves are kept small, then that will ensure a minimum consumption of hot and cold utilities.

SENSITIVITY TABLES

The Kotjabasakis and Linnhoff approach to determining the flexibility of a HEN uses sensitivity tables. These tables are constructed from the results of a simplified model of the HEN that only considers constant values of WC_p . Using this approach and applying the expected variation in T_S and WC_p , in a range from 50% to 100%, changes in UA (heat transfer area and overall heat transfer coefficient) and in the hot/cold utilities flow rate have to be made in order to keep the four target temperatures at the specified values. Table 10 shows these changes for maximum variations in the operating conditions. Although such changes require a cost evaluation they are larger to those found using design reliability theory. The advantage of using this approach is that it provides a warning of the extreme variations happening in the target temperatures. Although this approach will often suggest to the design engineer the conservative design, over-designing the hot and cold utilities, this approach also suggests that modification of the heat transfer areas can make the HEN more flexible. This will generally require additional expenditure compared to the results obtained using the design reliability approach.

5. Conclusions

Reliability/unreliability estimations proved to be a useful new tool to improve HEN design subject to uncertainties in the operating conditions. This technique helps to identify critical and sensitive areas of the HEN, through the inspection of the design constraint values for different possible variations in the operating conditions.

The HEN case study design was found to be less reliable to flow rate variations in the input streams than for supply temperature variations. Increasing

hot and cold utilities decreased the unreliability of the HEN design. Membership functions can be modified in order to control disturbances in some process streams. This can be achieved by using surge tanks or effective flow rate controllers upstream of the HEN, however such changes require a detailed cost analysis of all the alternatives. Due to process limitations, no control system will keep some target temperatures within the desired $\pm 5^\circ\text{C}$, if variations in heat capacity flow rate and supply temperature reach extreme values.

In conclusion, this new methodology for HEN analysis helps to determine the flexibility of the design and can be used to determine control strategies and especially alert about possible control limitations.

Acknowledgements

We thank Dr. Fred P. Stein, Emeritus Professor, Lehigh University for his helpful comments.

Hyprotech Ltd. is thanked for funding for Mr. R. Tellez.

References

- [1] Kubic, W.L. Jr and Stein, F.P., "A Theory of Design Reliability Using Probability and Fuzzy Sets", *AIChE Journal*, Vol. 34, No. 4, 583-601 (1988).
- [2] Türkay, M. and Grossmann, I.E., "Logic-Based MINLP Algorithms for the Optimal Synthesis of Process Networks", *Computers. Chem. Engng*, vol. 20, No. 8, 959-978 (1996).
- [3] Swaney, R.E. and Grossmann, I.E., "An Index for Operational Flexibility in Chemical Process Design. 1: Formulation and Theory", *AIChE Journal*, Vol. 31, No. 4, 621-630 (1985).
- [4] Bansal, V., Perkins, J.D. and Pistikopoulos, E.N., "Flexibility Analysis and Design of Linear Systems by Parametric Programming", *AIChE Journal*, Vol. 46, No. 2, 335-354 (2000).
- [5] Colberg, R.D., Morari, M. and Townsend, D.W., "A Resilience Target for Heat Exchanger Network Synthesis", *Computers Chem. Engng*, vol. 13, No. 7, 821-837 (1989).
- [6] Kotjabasakis, E. and Linnhoff, B., "Sensitivity Tables for the Design of Flexible Processes (1)-How Much Contingency in Heat Exchanger Networks is Cost-Effective?", *Chem. Eng. Res. Des.*, Vol. 64, 197-210 (1986).
- [7] Aguilera, N. and Marchetti J.L., "Optimizing and Controlling the Operation of Heat-Exchanger Networks", *AIChE Journal*, Vol. 44, No. 5, 1090-1104 (1998).
- [8] Ross, T.J., Booker, J.M. and Parkinson, W.J., "Fuzzy Logic and Probability Applications, Bridging the Gap", *ASA-SIAM Series on Statistics and Applied Probability* (2002).

[9] Zadeh, L. "Fuzzy Sets as a Basis for a Theory of Possibility", Fuzzy Sets Syst., Vol. 1, 3-28 (1978).

Nomenclature

A = Overall heat transfer area, m^2
 F = Set of operating conditions that violates a constraint
 g = Inequality design constraint, could be either $^{\circ}C$ or kg/s
 h = Equality design constraint, $^{\circ}C$
 R = Reliability
 s = Design constraint scaling factor, units similar to g_j
 T = Temperature, $^{\circ}C$
 U = Unreliability
 UA = Overall heat transfer coefficient/heat transfer area, $kW/^{\circ}C$
 WCp = Heat capacity flow rate, $W/^{\circ}C$

Greek letters

α = Lower bound value of all design constraints
 μ = Membership value

Subscripts

F = Denotes all sets of operating conditions that violate all constraints
 i = Number of fuzzy parameters
 j = Number of constraints
 S = Supply
 T = Target

Appendix

The following is a simple example showing how the design reliability calculations are carried out. For this example two fuzzy variables, p_1 and p_2 will be used with one degree of freedom u and the following four inequality constraints:

$$u \leq 2 \quad (A.1)$$

$$g_1 = (u - 0.25)^2 + (p_1 - 3.5)^2 - 15 \leq 0 \quad (A.2)$$

$$g_2 = (u - 1.25)^2 + (p_2 + 1.25)^2 - 1 \leq 0 \quad (A.3)$$

$$g_3 = (2.5u - 3.125) + (3p_1 - 1.5) - (2p_2 + 0.5) \leq 0 \quad (A.4)$$

The membership functions for p_1 and p_2 are shown in Figure A.1. Based on the above and if the initial nominal values of p_1 and p_2 are 0.3 and -0.5 respectively, the unreliability calculations yield the results shown in Table A1. For g_1 failure, there is no unique solution since both p_1 and p_2 affect u , but only p_1 determines the value of g_1 . Table A1 shows three possible solutions for g_1 failure. Therefore the maximum value of U^* has to be used. For g_2 failure, regardless of the value of p_1 , unreliability

calculations always yield the same value; therefore a unique value of unreliability is found. For g_3 , since the constraint is dependent on both p_1 and p_2 , there also is a unique solution. The values with asterisk come from calculations that were initiated using nominal values. The overall unreliability of this system is 0.84. In order to minimise the value of the unreliability and optimise the design, two alternatives are presented below.

MODIFYING CONSTRAINTS

This alternative involves the modification of the design constraints in order to improve the design.

$$g_1 = (u - 0.25)^2 + (p_1 - 3.5)^2 - 17 \leq 0 \quad (A.5)$$

$$g_2 = (u - 1.25)^2 + (p_2 + 1.25)^2 - 2 \leq 0 \quad (A.6)$$

$$g_3 = (2.5u - 4) + (3p_1 - 6) - (2p_2 + 2) \leq 0 \quad (A.7)$$

The unreliability results with these three new modified constraints are shown in Table A2. Again the algorithm does not guarantee to find the upper bound of the unreliability if departing from the nominal values of the fuzzy variables. Table A2 is only showing the results found from nominal values.

MODIFY MEMBERSHIP FUNCTIONS

This is another alternative; the supports of the membership functions can be shrunk as shown in Figure A.2. The unreliability for this case using the original constraints can be observed in Table A3. Again, there is no unique solution and no guarantee to find the upper bound of the unreliability if the algorithm departs from nominal values of the fuzzy variables.

Tables and Figures

Table 1. Design conditions.

Stream	WCp (kW/ $^{\circ}C$)	T _s ($^{\circ}C$)	T _T ($^{\circ}C$)
H1	30	300	80
H2	45	200	40
C3	40	40	180
C4	60	140	240

Table 2. UA average values.

Heat exchanger	A (m^2)
HX1	65.84
HX2	141.5
HX3	326.2
HX4	1504
CH1	17.01
CH2	60.35
HC4	7.88

Table 3. Fuzzy design parameters.

p_1	H1 supply temperature
p_2	H2 supply temperature
p_3	C3 supply temperature
p_4	C4 supply temperature
p_5	H1 heat capacity flow rate
p_6	H2 heat capacity flow rate
p_7	C3 heat capacity flow rate
p_8	C4 heat capacity flow rate

Table 4. Process design constraints.

	Constraint	Value
h_1	H1 target temperature	80 °C
h_2	H2 target temperature	40 °C
h_3	C4 target temperature	240 °C
g_1	Upper bound C3 target temperature	185 °C
g_2	Lower bound C3 target temperature	175 °C
g_3	Upper bound CW flow rate for CH1	25.1 kg/s (+25%) 30.2 kg/s (+50%)
g_4	Upper bound CW flow rate for CH2	16.2 kg/s (+25%) 19.5 kg/s (+50%) 22.7 kg/s (+75%)
g_5	Upper bound steam flow rate for HC4	1.26 kg/s (+25%) 1.51 kg/s (+50%) 1.76 kg/s (+75%)

Table 5. Design unreliability, with no changes in hot and cold utilities.

p	$U_j^*(\mathbf{u})$ (%)	Constraint
T_{S-H1} (+15 °C)	0.0	None
T_{S-H1} (-11.5 °C)	35.0	g_5
T_{S-H2} (+15 °C)	0.0	None
T_{S-H2} (-15 °C)	0.0	None
T_{S-C3} (+15 °C)	0.0	None
T_{S-C3} (-15 °C)	0.0	None
T_{S-C4} (+15 °C)	0.0	None
T_{S-C4} (-14.8 °C)	2.0	g_2
T_{S-C3} (-14 °C) and T_{S-C4} (-13.5 °C)	10.0	g_2
WCP_{H1} (+19.0%)	40.2	g_1
WCP_{H1} (-14.5%)	70.2	g_2
WCP_{H2} (+13.2%)	79.0	g_4
WCP_{H2} (-25.0%)	0.0	None
WCP_{C3} (+25.0%)	0.0	None
WCP_{C3} (-19.4%)	37.6	g_1
WCP_{C4} (+7.8%)	100.0	g_5
WCP_{C4} (-25%)	0.0	None
WCP_{H1} and WCP_{H2} (-9.2%)	100.0	g_5
WCP_{C3} and WCP_{C4} (-13.6%)	76.2	g_3
WCP_{H1} , WCP_{H2} , WCP_{C3} and WCP_{C4} (+18.6%)	42.4	g_5

Table 6. Design unreliability, with (+50%) changes in utilities.

p	$U_j^*(\mathbf{u})$ (%)	Constraint
T_{S-H1} (-15 °C)	0.0	None
WCP_{H1} (+19.0%)	40.2	g_1
WCP_{H1} (-14.5%)	70.2	g_2
WCP_{H2} (+20.8%)	28.2	g_4
WCP_{C4} (+15.8%)	61.6	g_5
WCP_{H1} and WCP_{H2} (-10.5%)	96.7	g_2
WCP_{C3} and WCP_{C4} (-21.2%)	25.6	g_4
WCP_{H1} , WCP_{H2} , WCP_{C3} and WCP_{C4} (+25.0%)	0.0	None

Table 7. Design unreliability, with (+75%) changes in utilities.

p	$U_j^*(u)$ (%)	Constraint
WCP_{H1} (+19.0%)	40.2	g_1
WCP_{H1} (-14.5%)	70.2	g_2
WCP_{H2} (+25.0%)	0.0	None
WCP_{C3} (-25.0%)	0.0	None
WCP_{C4} (+23.0%)	13.5	g_5
WCP_{H1} and WCP_{H2} (-10.5%)	96.7	g_2

Table 8. Design unreliability, with new membership functions and (+75%) changes in utilities.

p	$U_j^*(u)$ (%)	Constra
WCP_{H1} (+15.0%)	0.0	None
WCP_{H1} (-15.0%)	0.0	None
WCP_{H2} (+15.0%)	0.0	None
WCP_{H2} (-15%)	0.0	None
WCP_{C3} (+14.6%)	4.3	g_2
WCP_{C3} (-15.0%)	0.0	None
WCP_{C4} (+15.0%)	0.0	None
WCP_{C4} (-15.0%)	0.0	None
WCP_{H1} and WCP_{H2} (-15.0%)	0.0	None
WCP_{C3} and WCP_{C4} (-15.0%)	0.0	None
WCP_{C3} and WCP_{C4} (+15.0%)	0.0	None
WCP_{H1} , WCP_{H2} , WCP_{C3} and WCP_{C4} (+15.0%)	0.0	None

Table 9. HEN changes based on sensitivity tables approach

Maximum changes in operating conditions							Maximum modification
T_{S1}	T_{S2}	T_{S3}	T_{S4}	WCP_{H1}	WCP_{H2}	WCP_{C3}	WCP_{C4}
↑	↑	↑	↑	↑	↑	↓	↓
							UA for HX1 + 80%
							UA for HX2 + 20%
							CW for CH1 +130%
							CW for CH2 +175%
↓	↓	↓	↓	↓	↓	↑	↑
							UA for HX3 + 200%
							UA for HX4 + 200%
							steam for HC4 +330%

Table A1. Unreliability values for each constraint failure

When	g_1 fails			g_2 fails			g_3 fails
p_1	-	-	-	-	0.242	0.084	0.892*
	0.358*	0.306	0.420	0.300*			
p_2	-	-	-	-	-0.250	-0.25	-
	0.500*	0.291	1.000	0.250*			0.664*
u	0.589*	0.967	0.282	1.250*	1.250	1.25	0.44*
U^*	0.642*	0.694	0.000	0.625*	0.625	0.625	0.840*

Table A2. Unreliability values for each modified constraint failure

When	g_1 fails	g_2 fails	g_3 fails
p_1	-0.609	0.300	3.000
p_2	-0.500	0.000	-1.000
u	0.589	1.250	0.282
U^*	0.391	0.000	0.000

Table A3. Unreliability values for modified membership functions

When	g_1 fails	g_2 fails	g_3 fails
p_1	-0.358	0.300	0.899
p_2	-0.500	-0.294	-0.668
u	0.589	0.955	0.437
U^*	0.488	0.648	0.772

Fig. 1. Membership function for any operating condition.

Fig. 2. Heat exchanger network studied by Kotjabasakis and Linnhoff.

Fig. 3. Heat exchanger network with assignment of optimization variables.

Fig. 4. Membership functions for the supply temperatures.

Fig. 5. Membership functions for the heat capacity flow rates.

Fig. 6. Membership functions for the modified heat capacity flow rates.

Fig. 7. Variation in WCP C4 with respect to its membership function.

Fig. 8. Target temperature response with respect to supply temperature variations.

Fig. 9. Utility consumption with respect to supply temperature variations.

Fig. 10. Target temperature response with respect to flow rate variations.

Fig. 11. Utility consumption with respect to flow rate variations.

Fig. 12. MPC controller implemented on the heat exchanger network.

Fig. 13. Target temperature response with respect to supply temperature variations, using MPC.

Fig. 14. Utility consumption with respect to supply temperature variations, using MPC.

Fig. 15. Target temperature response with respect to flow rate variations, using MPC.

Fig. 16. Utility consumption with respect to flow rate variations, using MPC.

Fig. A1. Membership functions for p_1 and p_2 .

Fig. A2. Modified membership functions for p_1 and p_2 .