A Formal Design Science

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Abstract

This paper presents the framework of a formal design science through summarizing and reviewing the author's research of design over the last decade. The paper firstly defines the scope of design science: nature-designer system. The reasoning capability of design science is developed on the basis of five axioms proposed by the author. Some theorems of design are briefly derived from these axioms. An industrial application of this formal design science is also presented. All these results are in the framework defined by the nature-designer system. Future directions are also given in the paper.

Key Words—*design science, scope of design science, reasoning capability, theorems of design, recursive logic*

1 Introduction

Over the last two decades, many computer-aided design software systems have become available in the market, such as Pro/Engineer and Solidworks. These software systems have greatly benefited manufacturing industries. Majority of these systems focus on modeling the geometric information appearing in the design process. Hence, their major functionality is on assisting product detailed design. Encouraged by the success of these systems, researchers have been attempting to develop computer-aided conceptual design tools [1]. However, the efforts are not as successful as geometric modeling tools. This can be partly explained by the following observations: first, design needs analysis. In the history of product design, most of innovative and complex products come from our ability to logically analyze and predict the performances of the products to be designed based on scientific knowledge. The success of current CAD systems for detail design, such as Pro/Engineer and Solidworks, is such an example. They are built up on the basis of systematic knowledge of geometry. Secondly, to analyze complex design needs a science founded on logic. Both scientific principles and logical reasoning are indispensable to do the analysis, as can be seen in most of existing engineering sciences. Finally, there is no such a science in the literature that can fully logically analyze design activities. Most of design theories are established from observations and experimentation [2-6]. As a result, only simple prototypes of computer-aided conceptual design systems are reported in the literature.

Design science should consist of two different but strongly interconnected realms: a system of logically related knowledge about design activities as well as the scientific

processes leading to the knowledge [7, 8]. The scientific processes encompass observation, experimentation, and reasoning. Though observation and experimentation have been widely used in design research, reasoning (especially the mathematical reasoning) has not become a widely available tool in the design research community. However, reasoning is the most primary scientific process. The reasoning ability provides the capacity to analyze design activities through extrapolating existing knowledge to new situations. It also provides possibilities for developing sophisticated and robust computer-aided conceptual design systems. In addition, it has the potential to enhance existing design methodologies [3-6].

Formal design science is the design science whose object of study is inclusively represented in mathematics and whose reasoning follows the logic underlying mathematics. The difference of formal design science from other types of design science (e.g. [7, 8]) mainly lies in the tool to represent its object of study. The formal design science attempts to use mathematics while others use informal media such as natural language, graphic illustrations, and flow charts. This paper presents the author's research results from the efforts in developing such a science over the last decade.

Formal design science is such a new subject that its scope, objectives, approaches, and knowledge have not been well defined [8]. The rest of this paper consists of four major sections: scope of design science, mathematical reasoning capability, theorems about design activities, and applications. The last section concludes this paper and gives some future research problems.

2 Scope of Design Science

Generally speaking, any endeavor to understand design can be seen as an effort to expand the design science in one way or another. In the literature, some endeavors focus on product such as [9, 10] while some others focus on design process [3, 4, 11]. The most comprehensive description of the object of design science found in the literature is design activities [7]. But the scope and contents of design activities were not clearly defined. This paper will look into the object of study of design science from object and process perspectives. From the perspective of object, the object of study of design science includes designer, product, environment, and their mutual relations. This is graphically given in Figure 1.

From the perspective of process, the object of study of design includes problem formulation, conceptual design,

embodiment design, and detail design [5]. This is graphically illustrated in Figure 2.

Figure 1. Object of study of formal design science: arrows represent relationships

Figure 2. Basic design process

One challenge in developing such a formal science is to equip it with a reasoning ability for analyzing design activities. This challenge is manifested in two folds: first, designer, product, and environment can have components, which makes the full representation of the objects in Figure 1 hierarchical [12]. Second, the design process in Figure 2 is an iterative process, in which designers may add new components as well as modify or delete existing components in a random way. These two aspects together make the object of design dynamic and hierarchical. A formal design science needs to provide a mathematical tool that has a well-defined hierarchical *structure* while it can still *flexibly* represent and reason about the dynamic hierarchical structure of design.

Another challenge in developing such a formal science is to model designers in a formal mathematical framework.

The third challenge is how to derive theorems about design activities and solve design problems using the formal design science.

The next sections will address these three challenges.

3 Reasoning Capability of Design Science

The reasoning capability of a formal science usually comes from two aspects:

- 1) a mathematical theory that is able to represent and operate on the object of study as well as derived theorems of this science.
- 2) fundamental assumptions or axioms that provide necessary and sufficient number of premises to derive

knowledge about the relations within the object of study of this science.

3.1 Mathematical Foundation

In the literature, most of efforts in building up a mathematical theory for design have been based on set theory. These include Yoshikawa and Tomiyama [11, 13], Salustri and Venter [14], Braha and Maimon [15], Zeng and Gu [16]. The major difference among them lies in the proportion of the dynamic hierarchical structure that has been captured and represented. In spite of the success of representing the hierarchical structure of design, all these efforts remain to be able to only represent the knowledge about design activities without the capability to reason about them following a logical manner. The dynamic characteristic of the hierarchical structure is not supported, because two structures representing two respective states of the design process can be logically inconsistent due to the membership relation in set theory [17]. An alternative to this representation is mereology where no commitment is made to the existence of abstract entities [18]. It provides a way to put all product information in a logically consistent mathematical representation [10]. However, the hierarchical structure is lost [19]. A new mathematical representation of the dynamic hierarchical structure is developed by Zeng in the axiomatic theory of design modeling [17]. This theory provides a formal approach that allows for the development of design theories following logical steps based on mathematical concepts and axioms. This subsection will review the basic concepts of its mathematical foundation and introduces some new development.

Axiomatic theory of design modeling uses the following primitive concepts: universe, object, and relation. The following informal definitions are adapted from the Random House Webster's Unabridged Dictionary.

[Definition 1] The universe is the whole body of things and phenomena observed or postulated.

[Definition 2] An object is anything that can be observed or postulated in the universe.

It can be seen from these two definitions above that universe is the whole body of objects.

[Definition 3] A relation is an aspect or quality that connects two or more objects as being or belonging or working together or as being of the same kind. Relation can also be a property that holds between an ordered pair of objects.

$$R = A \sim B, \exists A, B, R, \quad (1)$$

where A and B are objects. $A \sim B$ is read as “ A relates to B ”. R is the relation from object A to object B . A relation of one object to itself is called the relation on the object itself.

Definition 3 has indeed given four major types of relations in the universe: being, belonging, working together, and being of the same kind. These relations may embody one or more of the basic properties of relations below:

- 1) Idempotent

$$R = A \sim A, \forall A. \quad (2)$$

2) Commutative
 $R = A \sim B = B \sim A, \exists A, B, R.$ (3)

3) Transitive
 $R = A \sim C$ if $R = A \sim B = B \sim C, \exists A, B, C, R.$ (4)

4) Associative
 $(A \sim B) \sim C = A \sim (B \sim C), \exists A, B, C.$ (5)

5) Distributive
 $(A \sim_1 B) \sim_2 C = (A \sim_2 C) \sim_1 (B \sim_2 C),$ (6)
 $\exists A, B, C, \sim_1, \sim_2.$

Based on these concepts, two axioms can be defined in the axiomatic theory of design modeling.

[Axiom1] Everything in the universe is an object.

[Axiom 2] There are relations between objects.

It can be seen from these two axioms that the characteristics of relations would play a critical role in the axiomatic theory of design modeling. We need to define a group of basic relations to capture the nature of object representation. There are many types of relations, such as being, belonging, working together, and being of the same kind given in Definition 3. To represent various relations in the universe, a group of basic relations are indispensable. This group of basic relations includes the corollaries of the axiomatic theory of design modeling. Two such corollaries have been developed in the theory. More are under development based on the needs of design research:

[Corollary 1] Every object in the universe includes other objects. Symbolically,
 $A \supseteq B, \forall A \exists B.$ (7)

B is called a subobject of A. The symbol $\supseteq$ is inclusion relation. The inclusion relation is transitive and idempotent but not commutative. Other operations such as $=$ (identity), $\cup$ (union), $\cap$ (intersection), and $-$ (difference) can be defined based on this corollary [17].

Mereology is indeed established based on this corollary [18]. Therefore, all conclusions in mereology can be adapted into our theory.

[Corollary 2] Every object in the universe interacts with other objects. Symbolically,
 $C = A \otimes B, \forall A, B \exists C.$ (8)

C is called the interaction of A on B. The symbol $\otimes$ represents interaction relation. Interaction relation is idempotent but not transitive or associative. The following rules hold for interaction relations:

$$\begin{aligned} A \otimes (B \cup C) &= (A \otimes B) \cup (A \otimes C), \\ (A \cup B) \otimes C &= (A \otimes C) \cup (B \otimes C), \\ A \otimes (B \cap C) &= (A \otimes B) \cap (A \otimes C), \\ (A \cap B) \otimes C &= (A \otimes C) \cap (B \otimes C). \end{aligned} \quad (9)$$

Based on Corollary 1 and 2, the structure operation can be developed as follows:

[Definition 4] *Structure operation*, denoted by $\oplus$, is defined by the union of an object and the relation of the object to itself.

$$\oplus O = O \cup (O \otimes O). \quad (10)$$

where $\oplus O$ is the structure of object O.

The structure operation provides the aggregation mechanism for representing the object evolution in the design process.

Based on structure operation and other relations, we can also define a new operation: separation operation Θ , as follows:

[Definition 5] For any object A, the operation to separate its sub-object B is called separation operation, denoted by Θ .

$$A \Theta B = (\oplus A) - (\oplus B), \forall B \subseteq A. \quad (11)$$

It should be noted that the symbol Θ was used to represent another operation in [17]. However, that operation was abandoned in the evolution process of this theory. It can be seen from the rest of this paper that the structure operation $\oplus$ and the separation operation Θ provide a logical tool for reasoning about design activities.

3.2 Representing Design Activities

As can be seen in Figure 1, design activities involve three objects: designer, product, and environment. Using structure operation, we can define engineering system and nature-designer system.

[Definition 6] An *engineering system* is the structure of an object (Ω) including both product (S) and its environment (E).
 $\oplus \Omega = \oplus (E \cup S) = (\oplus E) \cup (\oplus S) \cup (E \otimes S) \cup (S \otimes E).$ (12)

Generally speaking, everything except the product itself can be seen as its environment. There are direct, close, and remote environments [6].

Figure 3. Engineering system

An engineering system can be illustrated in Figure 3. Both $E \otimes S$ and $S \otimes E$ together are called product-environment boundary (B). There are two types of product-environment boundary: structural boundary and physical interactions. The structural boundary is the shared physical structure between the product and its environment. The physical interactions include actions ($E \otimes S$) of the environment on the product and responses ($S \otimes E$) of the product to the environment. Therefore,
 $B = (E \otimes S) \cup (S \otimes E).$ (13)

Figure 4. State of design

$\oplus E$ and $\oplus S$ are structures of the environment and product. Since both product and environment may have components, $\oplus E$ and $\oplus S$ can be further decomposed into the structures of

these components as well as their mutual relations. This provides a recursive hierarchical representation of design. This representation has indeed provided a base for representing the state of design [20]. This is shown in Figure 4. A more comprehensive study of this recursive hierarchical structure is given in [17, 21].

Indeed, the engineering system is a natural system if no restriction is made on its environment. Hence, the object of design science shown in Figure 1 is equivalent to the nature-designer system defined as follow:

[Definition 7] A *nature-designer system* is the structure of an object (Ψ) including both nature (Ω) and designer (M).

$$\oplus \Psi = \oplus (\Omega \cup M) = (\oplus \Omega) \cup (\oplus M) \cup (\Omega \otimes M) \cup (M \otimes \Omega). \quad (14)$$

The nature-designer system is graphically illustrated in Figure 5. Depending on the context of research, the focus on designer can be the designer's thinking process, cognitive process, etc. Eekel has arrived at a similar description as the foundation of his design research [8]. The difference lies in that Eekel developed the diagram based on his experience whereas this paper derives this diagram logically from more fundamental axioms.

Figure 5. Nature-designer system

3.3 Representing Designers

Since our focus is on the design activities, the designer's thought process is our major concern. The structure of the designer's thought is called human rational system. The following defines four relations in the nature-designer system:

[Definition 8] Natural law L is a sub-object of the relation $\Omega \otimes \Omega$, and $L \subseteq \Omega \otimes \Omega$. (15)

[Definition 9] Knowledge K is a sub-object of the relation $M \otimes M$, and $K \subseteq M \otimes M$. (16)

[Definition 10] Recognition is a relation f^R from nature Ω to the human thought M , and $f^R \subseteq \Omega \otimes M$. (17)

[Definition 11] Action is a relation f^A from the human thought M to nature Ω , which influences nature Ω by implementing a plan or a design produced by the human thought M , and $f^A \subseteq M \otimes \Omega$. (18)

Design is an activity happening in the human rational system. The results have to be sent back to the natural system through action. Once a design is materialized in nature, it has to follow natural laws. Whether and how a design can survive in nature depends on the answers to the following questions:

1. What is the character of nature?

2. What is the character of recognition?

3. What is the character of the human rational system?

The answer to the first question can be philosophical and theological, for it depends on how human beings understand nature. Any answer to this question would only be relatively true with respect to the answers to the last two questions, since it is itself a result from human recognition and reasoning. In accordance to the commonsense, three axioms are developed to address the last two questions.

First, according to Definition 4, the structure of an object depends on what objects are included in the object. As we can see in Figure 5 and Definition 10, the goal of the recognition process is to define the structure of an object in nature with the object(s) in the human thought. If the human thought has a one-to-one correspondence to nature, then they would have perfect knowledge. This is the axiom of correspondence in Yoshikawa's General Design Theory [11]. But it is against the human commonsense. The axioms of bounded rationality and recognition address this character.

[Axiom 3] Human beings are bounded in rationality.

This axiom is adopted from Simon [12]. In this axiom, rationality is the quality or state of being having reason or understanding. The main manifestation of this axiom is the limitation of resources (e.g., time and memory) in the rational system. A direct result is the limited number of objects in the human thought M . That is

$$M = \bigcup_{i=1}^n O_i^a, \quad (19)$$

where i and n are finite natural numbers. Each object O_i^a is called a primitive object.

Substituting Equation (19) into Equation (16),

$$K \subseteq \left(\bigcup_{i=1}^n O_i^a \right) \otimes \left(\bigcup_{i=1}^n O_i^a \right). \quad (20)$$

Since n is a finite natural number, knowledge K is limited. This means that the amount of human knowledge is limited.

Figure 6. Human recognition process

This axiom is represented in Figure 6. The choosing of primitive objects is artificial. It can be observable and/or measurable properties, or even objects with complex structure. One of the tasks of scientific research is to look for the minimum number of primitive objects to describe various natural phenomena.

Another issue in the human recognition process is the nature of recognition. This is addressed by the following axiom:

[Axiom 4] Human beings do not recognize objects accurately. That is,

$$(O' = f^R(O)) \wedge (O' \neq O), \forall O \subseteq N \exists O' \subseteq M. \quad (21)$$

This axiom has brought in the ambiguity of design problem

formulation and design knowledge, which further leads to creative design [20].

Second, let us consider a special case where an object O in the human rational system consists of three subobjects: cause O^c , object O^a , and effect O^e , which is shown in Figure 7.

Figure 7. Object consisting of three subobjects

This object is symbolically represented as

$$O = O^c \cup O^a \cup O^e, \text{ for } O, O^c, O^a, O^e \subseteq M. \quad (22)$$

Since the formal definitions of cause and effect are rather philosophical [22], we will not identify their semantic differences in this paper. This treatment does not affect the present discussion. In terms of (16), knowledge K for this object can be represented as

$$\begin{aligned} K \subseteq O \otimes O &= (O^c \cup O^a \cup O^e) \otimes (O^c \cup O^a \cup O^e) \\ &= (O^c \otimes O^c) \cup (O^a \otimes O^a) \cup (O^e \otimes O^e) \cup \\ &\quad (O^c \otimes O^a) \cup (O^a \otimes O^e) \cup (O^c \otimes O^e) \cup \\ &\quad (O^e \otimes O^a) \cup (O^a \otimes O^c) \cup (O^e \otimes O^c). \end{aligned} \quad (23)$$

The relations in Equation (23) can be divided into three groups:

1. $O^c \otimes O^c, O^a \otimes O^a, O^e \otimes O^e$.
2. $(O^c \otimes O^a) \cup (O^a \otimes O^e) \cup (O^c \otimes O^e)$.
3. $(O^e \otimes O^a) \cup (O^a \otimes O^c) \cup (O^e \otimes O^c)$.

The three relations in the first group include knowledge about each object itself. The relations in the second and third groups lead to the inductive knowledge regarding the object O^a : $O^c \otimes O^e$ or $O^e \otimes O^c$.

To address the difference between $O^c \otimes O^e$ and $O^e \otimes O^c$, the causal relation is defined in the following manner:

[Definition 12] The causal relation is the relation from cause to effect in the human rational system.

[Axiom 5] The causal relation is the only deterministic relation in all relations between causes and effects.

$$e_c = C^r(e_c), \forall e_c \subseteq O^c \subseteq M, \exists e_e \subseteq O^e \subseteq M, \exists C^r \subseteq O^c \otimes O^e, \quad (24)$$

where C^r is a causal relation between causes O^c and effects O^e . This axiom determines that human reasoning is not symmetric. Only deductive reasoning is deterministic and all other reasoning modes are plausible.

To capture the designer's way of thinking, Zeng and Cheng [16] proposed a new logic of design, which is named recursive logic. This logic states that design problem solving is a process recursively generating design solutions and the knowledge to evaluate the solutions. A comparable conclusion was also reached by Roozenburg [23, 24]. Maher [2] later developed a design process model called co-evolution to capture the same nature.

4 Knowledge About Design Activities

The reasoning capability is only the core of a science. The systematic knowledge about its object of study is the contents that make the science useful. This section derives some knowledge about design activities using this reasoning capability. This includes theorems about the structure of design problem and product as well as the nature of design problem (design governing equation). The details of all these theorems are addressed in separate papers. These theorems are just a few examples derived in this science. Other theorems regarding the design process, design creativity, conflict resolution, etc are also developed from the axioms in this science.

4.1 Theorem of Design Problem Structure

Representing various design problems in a well-defined structure, if possible, would be beneficial for solving design problems and modeling the solving process. This section presents a theorem we have derived about the structure of design problems using the axiomatic theory of design modeling.

Before we derive the structure of design problem, the following lemma should be introduced:

[Lemma 1] For four objects A_1, A_2, B_1 , and B_2 , $A_i \cap B_j = \Phi$ ($i, j=1, 2$). If $(A_1 \cup B_1) \supseteq (A_2 \cup B_2)$, then $A_1 \supseteq A_2$ and $B_1 \supseteq B_2$. Φ is a null object which is included in any object.

Informally, a *design problem* can be defined as a proposition in which something has to be designed to meet the descriptions of a request for the design. In this statement, "something to be designed" can be seen as the final design solution of the design problem, which is an engineering system. This system can be formally represented as $\oplus \Omega_s$ ($\Omega_s = E_s \cup S_s$), where E_s and S_s are the environment and product by the end of the design process, respectively. On the other hand, "the descriptions of a request for the design" can also be seen as an engineering system for which the product is not or only partially defined. Denoting the corresponding environment and product by E_0 and S_0 , respectively, we can formally define this engineering system as $\oplus \Omega_0$ ($\Omega_0 = E_0 \cup S_0$). In the stage of formulating a design problem, $\oplus \Omega_s$ is an unknown and $\oplus \Omega_0$ is the only thing defined. Obviously, $\oplus \Omega_s$ should include all the information in $\oplus \Omega_0$. Therefore, a design problem can be symbolically represented as,

$$P^d = \lambda(\oplus \Omega_s, \oplus \Omega_0), \quad (25)$$

where λ is the inclusion relation ($\supseteq$). A design problem is solved when P^d assumes the value of "true".

According to (12) and (13), we have

$$\oplus \Omega_s = (\oplus E_s) \cup (\oplus S_s) \cup B_s, \quad (26)$$

$$\oplus \Omega_0 = (\oplus E_0) \cup (\oplus S_0) \cup B_0.$$

Since $E_i \cap S_j = \Phi, \forall i, j = s, 0$, according to Lemma 1, we have

$$P^d = \lambda(\oplus E_s, \oplus E_0) \wedge \lambda(\oplus S_s, \oplus S_0) \wedge \lambda(B_s, B_0). \quad (27)$$

This equation has been used to study the mechanism behind creative design [20].

5 Industrial Application

In addition to understanding design activities, the proposed formal design science has also been applied to develop tools for assisting the design process. One application is the formal specification of user requirements [33]. This section presents an application in sketch-based conceptual design [21].

An important issue in developing a sketch-based conceptual design system is the reconstruction of 3D model from 2D line drawings. Two objects are involved in this 3D reconstruction process: 2D line drawing and 3D topology. They can be denoted by G^2 and G^3 , respectively. Then the structure Σ constituting these two objects can be represented as

$$\Sigma = \oplus(G^2 \cup G^3). \quad (38)$$

By applying (10), we have

$$\Sigma = (\oplus G^2) \cup (\oplus G^3) \cup (G^2 \otimes G^3) \cup (G^3 \otimes G^2). \quad (39)$$

Graphically, (39) can be illustrated in Figure 9.

As was stated in [34], "the goal of the sketch reconstruction is to restore the original 3D object using the information derived from the projection only". The input of this algorithm is a line drawing including the information of faces, edges, and vertices, etc. The z-coordinates of all the vertices in the line drawing are zero.

Based on this problem statement, the input is:

$$\text{Input} = (\oplus G^2) \cup (G^3 \otimes G^2) \quad (40)$$

while the output should be

$$\text{Output} = (\oplus G^3) \cup (G^2 \otimes G^3). \quad (41)$$

Figure 9. Graphic illustration of sketch interpretation

$\oplus G^2$ is the structure of the 2D line drawing. It includes two parts: first, the basic geometric information (G^2), such as faces, lines, as well as vertices; secondly, the relation ($G^2 \otimes G^2$) between geometric elements included in G^2 , such as the angle between two straight lines, the length of an edge, the shape of a face, etc.

The relation $G^3 \otimes G^2$ represents the projection from 3D object to 2D plane. This is a causal relation. Each point $P(x,y,z)$ in G^3 is projected into $P'(x',y')$ in G^2 . This projection relation can be represented as

$$x' = x, y' = y. \quad (42)$$

Here it is assumed that the projection ray is perpendicular to the projection plane. Obviously, the z-value is lost in the projection process.

$\oplus G^3$ is the structure of the 3D geometry. It includes two

parts: first, the basic geometric information (G^3), such as 3D components, faces, lines, as well as vertices; secondly, the relation ($G^3 \otimes G^3$) between geometric elements included in G^3 , such as the angle between two straight lines or two planes, the length of an edge, the area of a region, the shape of a face, etc.

The relation $G^2 \otimes G^3$ represents the reconstruction of 3D information of the object from its 2D projection. This relation is plausible according to Axiom 5. The z-value of vertex corresponding to each point $P'(x',y')$ in G^2 has to be found.

The purpose of the algorithm is to define the relation $G^2 \otimes G^3$ so that G^3 can be constructed and therefore the structure Σ becomes well-defined.

There are many approaches for finding the z-values based on a 2D line drawing. This mainly depends on the selection of primitive objects for the 2D line drawing and 3D topology. This is guaranteed by Axiom 3.

Suppose that there exists a collection of primitive geometric objects G_a^2 in G^2

$$G_a^2 \subset G^2. \quad (43)$$

Therefore, a 2D line drawing, g , can be decomposed into a collection of primitive geometric objects such that

$$g = \bigcup_{i=1}^n g_i \quad \forall g \subset G^2 \quad \exists g_i \subset G_a^2 \quad (i=1,2,\dots,n). \quad (44)$$

Furthermore, suppose that there exists a collection of primitive geometric objects G_a^3 in G^3

$$G_a^3 \subset G^3. \quad (45)$$

Therefore, a 3D topology η can be decomposed into a collection of primitive geometric objects such that

$$\eta = \bigcup_{j=1}^m \eta_j \quad \forall \eta \subset G^3 \quad \exists \eta_j \subset G_a^3 \quad (j=1,2,\dots,m). \quad (46)$$

Denote the geometric knowledge about 2D and 3D geometries by K_{g2} and K_{g3} , respectively, then

$$\begin{aligned} K_{g2} &\subset G_a^2 \otimes G_a^2, \\ K_{g3} &\subset G_a^3 \otimes G_a^3. \end{aligned} \quad (47)$$

Figure 10. 2D line drawing of a cubic

As a result, $\bigoplus \bigcup_{i=1}^n g_i$ defines the 2D line drawing in terms of primitive objects and their relations. For the example of 2D line drawing in Figure 10, there are many ways to define the primitive objects. Examples include edges (L_i), faces (F_i), and

vertices (V_i). If we define the drawing in terms of edges, the relation $(\bigcup_{i=1}^n g_i) \otimes (\bigcup_{i=1}^n g_i)$ includes $L_5 // L_6$, $L_1 \perp L_5$, etc. If we define the drawing in terms of faces, the relation $(\bigcup_{i=1}^n g_i) \otimes (\bigcup_{i=1}^n g_i)$ includes “ F_1 and F_2 are neighbors”, “ F_1 , F_2 , and F_3 are three visible faces”, etc. If we take vertices as primitives, the relation $(\bigcup_{i=1}^n g_i) \otimes (\bigcup_{i=1}^n g_i)$ would include “ V_3 and V_4 are connected”, “The angle $\angle V_2 V_4 V_3$ is 90° ”.

$\bigoplus \bigcup_{i=1}^n \eta_i$ defines the 3D topology of an object in terms of primitive objects and their relations. Primitive objects for a 3D object can also be edges, faces, and vertices. The difference of a 3D geometry from a 2D line drawing lies in that vertices have their z-values defined. Hence,

$$\bigoplus g = \bigcup_{i=1}^n g_i \cup \bigcup_{i=1}^n \bigcup_{j=1}^n (g_i \otimes g_j) \subset \bigcup_{i=1}^n g_i \cup K_{g2}. \quad (48)$$

The same will be true for 3D topology:

$$\bigoplus \eta = \bigcup_{i=1}^m \eta_i \cup \bigcup_{i=1}^m \bigcup_{j=1}^m (\eta_i \otimes \eta_j) \subset \bigcup_{i=1}^m \eta_i \cup K_{g3}. \quad (49)$$

Furthermore, we can denote the projection and reconstruction relations as $\wp$ and C , respectively. Hence,

$$\wp = (\bigcup_{j=1}^m \eta_j) \otimes (\bigcup_{i=1}^n g_i), \quad C = (\bigcup_{i=1}^n g_i) \otimes (\bigcup_{j=1}^m \eta_j). \quad (50)$$

Since (44) and (46) are for any object in their corresponding contexts, they can be substituted into (39). Considering (48) and (49), we have

$$\begin{aligned} \Sigma &= (\bigoplus_{i=1}^n g_i) \cup (\bigoplus_{j=1}^m \eta_j) \cup (\bigcup_{i=1}^n g_i \otimes \bigcup_{j=1}^m \eta_j) \cup (\bigcup_{j=1}^m \eta_j \otimes \bigcup_{i=1}^n g_i) \\ &\subset (\bigcup_{i=1}^n g_i) \cup (\bigcup_{j=1}^m \eta_j) \cup K_{g2} \cup K_{g3} \cup C \cup \wp. \end{aligned} \quad (51)$$

To choose the primitive objects, let us look at the relation $\wp$ of the projection from 3D geometry to 2D line drawing. This relation is causal and deterministic, under which a unique 2D representation of a 3D object will be produced. This projection relation can be further expressed as follows:

$$\wp = (\bigcup_{j=1}^m \eta_j) \otimes (\bigcup_{i=1}^n g_i) = \bigcup_{j=1}^m \bigcup_{i=1}^n (\eta_j \otimes g_i). \quad (52)$$

Eq. (52) defines a group of projection knowledge, based on which a collection of primitive reconstruction relation can be developed as follow:

$$C^a = \bigcup_{i=1}^n c_i^a, \quad \forall g_i \exists c_i^a \exists \eta_j [c_i^a = g_i \otimes \eta_j]. \quad (53)$$

A recursive algorithm is developed to make use of these primitive reconstruction relations to reconstruct 3D topology of an object [21]. The following gives two examples from a software prototype we developed based on this algorithm. Figure 11a) is a 2D line drawing of a single component where z-values of all vertices are not known. Figure 11b) is the 3D

topology corresponding to Figure 11a). Hidden faces are not processed. Figure 12a) is a 2D line drawing of an assembly of three components where z-values of all vertices are not known. Figure 12b) is the 3D topology corresponding to Figure 12a).

Figure 11. a) is the input of the problem where the z-values of all vertices are zero. b) is the 3d reconstruction of a). It shows the back of the model. Hidden faces are not reconstructed.

Figure 12. a) is the input of the problem where the z-values of all vertices are zero. b) is the 3d reconstruction of a). It shows the rotated model. Hidden faces are not reconstructed.

6 Conclusion

This paper established a formal design science by summarizing and reviewing the author's research on design over the last decade. The foundation of this design science includes the recursive logic of design [16] and the axiomatic theory of design modeling [17]. The elements in the recursive logic have been the object of modeling of the design science [20, 21, 25-27, 30, 33, 35, 36].

This paper proposes that the scope of design science is the nature-design system, which can be formally represented using the structure operation in this science. Five axioms in the formal design science provide a reasoning capability for deriving theorems about design activities. These five axioms are divided into two groups: axioms of objects and axioms of the human thought. Axioms of objects state that everything in the universe is an object, and that there are relations between objects. They provide the premise for studying objects in both nature and the human thought. Axioms of the human thought identify the nature of the reasoning and recognition processes in the human thought. They state that human beings are bounded in rationality, that human beings do not recognize objects accurately, and that causal relation is the only plausible relation among all relations between causes and effects.

An industrial application of this formal design science is presented to show its usefulness [21]. This science has also been applied to formalize user requirements [33] and study design creativity [20].

The significance of this work lies in that it provides a

logical method for approaching design studies. With this approach, design studies become a scientific exploration through the derivation of mathematical equations plus the explanation of the factual meaning of these equations. This exploration could lead to results that are already known from other approaches. Meanwhile, it also opens the possibility that some new results may be found.

The future work of this research includes the development of more operations for studying design activities and the formalization of existing design theory and methodology such as [3-6]. Experimental study is also important to validate the theorems derived from this science. More industrial applications will also be explored.

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