Effective Stiffening of Thin Rotating Disks using Shape Memory Alloys

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Abstract

Thin spinning disk such as saw blades are limited in their rotation speed by their critical speed. The critical speed can be lowered by heating along the periphery of the disk. To counter this phenomenon, the use of shape memory alloys is investigated as a method to stiffen the disk when heated. The disk is modeled using the finite element analysis. The design results in a thin disk that can operate at higher temperatures and speeds.

1. Introduction

Thin spinning disk are limited in their rotation speed by the critical speed. The speed at which the backwards travelling wave becomes standing when observed by a stationary observer. These critical speed limits the speed of operation of equipment such as saw blades, turbines and refiner plates used in mechanical pulping. [1, 2, 3] Although equipment such as turbines can be operated faster than critical speed, other equipment such as saw blades are largely limited as they do not operate in steady state conditions.

Thermal stresses in disks as a result of heating (blade-workpiece interaction, steam) can cause the critical speed of the disk to lower. The reduction in critical speed results in limitation in operating speed, and the need to design larger disks. [4, 5]

The authors have proposed a solution to the problem of thermal stresses by incorporating shape memory alloy inserts into the disks. These SMA inserts would induce tensile stresses into the periphery of the disk in proportion to the compressive stresses resulting from heating.

2. Disk Theory

Spinning disk are limited by their critical speed. The critical speed of the disk is the angular velocity at which a disk will become unstable. In disks where the vibration is a function of both bending and membrane stresses are present, the critical speed can be found by

the following equation. Where n is the number of nodal diameters for the lowest mode of vibration. (Eqn. 1)

Equation 1.

$$\omega_{cr} = \omega_0 = \frac{\omega_n}{n}$$

The frequency of vibration has been found to be the sum of the squares of the bending and membrane vibrations of the disk. [4] Therefore the disks are subject to changes in membrane stress in the disk, such as rotation stresses and thermal gradients. The stresses due to rotation increase the critical speed of the disk. A thermal gradient of thin disk which is heated at the outer periphery of the disk will result in compressive hoop stresses near the edge of the disk and results in a decrease in the critical speed of the disk. To compensate for this, current methods include prestressing the disk using plastic deformation and heating the disk at an inner radius, allowing the disk to expand freely. [6]

3. SMA Theory

Shape memory alloys are a class of metals that can exhibit shape memory. In the low temperature state, the martensite phase, the material is easily deformed into a new shape. When heated fully into the austenite phase, the material will revert back to its predeformed shape. This is the shape memory effect. It is usually a one way mechanism, such that cooling the material will not necessary reverse the shape transformation. If constrained the material will produce large recovery stresses. [7]

SMA has been used in active vibration applications such as, beams and helicopter blades. In this application, wires of SMA are imbedded into the material under a deformed condition. To control the vibration the wires are heated altering the state of stress in the beam, reducing the vibration trough damping and increasing the frequency of vibration. [7] These

systems are active systems and require outside controllers.

The authors proposed design utilises the heat generated by the cutting action to activate the shape memory alloy. Constrained on the steel disk the SMA will introduce tensile hoop stress into the disk in proportion to the disk heating.

The shape memory effect is based on the phase change from the martensite phase to the austenite phase. In the martensite phase, the material is easily deformed due to lattice slippage. When heated into the austenite phase the material will return to its memorised shape. The memorised shape in the material can be obtained by constraining the material and heating in the range of 400°C to 600°C for about an hour.

When a deformed piece of SMA is constrained during the martensite to austenite transformation stage the material will produce constraint force. For Nickel Titanium (NiTi) the forces can range from 200 MPa to 1400 MPa. Higher stresses reduce the fatigue life of the SMA. For lower constraint stresses of 200 MPa, the material is capable of cycles in excess of 100,000. [7]

4. Design

The saw blade was simulated using AnsysTM, a Finite element analysis program, FEA. The saw blade was simulated as a disk. The materials used were steel and NiTi (50.4%).

The shape memory effect was simulated in the FEA by altering the NiTi coefficient of thermal expansion to fit the recovery displacement of the NiTi. The martensite to austenite transformation is assumed to be linear between the Austenite start temperature and the austenite end temperature. These properties were not simulated. Outside the transformation temperatures the SMA has a constant coefficient of thermal expansion as given in literature. Figure shows the coefficient of thermal expansion relative to 0°C.

Figure 1. Values of thermal coefficient of expansion used to model SMA

The Young's modulus of the SMA also changes during the phase change. It is assumed to be linear during the phase change, going from 40 GPa at the T_{As} to 80 GPa at T_{Af} .

Two disks were simulated. A steel disk 5mm thick with a radius of 300 mm and a composite disk of the same dimensions. The composite disk is composed of steel and SMA. From $r=.25$ the clamping radius (r_A) to $r=.8$ the disk is entirely steel. From $r=.8$ to 1.0 the disk is composed of two SMA layers 1 mm thick sandwiching the steel disk 3mm thick. The two bands of SMA were deformed in the martensite phase 2 % in the hoop direction prior to be attached. When heated to T_{Af} the SMA will fully regain the 2 % deformation. The SMA is assumed to be perfectly bonded to the steel.

The following boundary conditions were applied. The disk is clamped at r_A and is free at r_B . The disk is subject to heating at a constant temperature at r_B . The disk is subject to cooling by convective heat transfer from both sides of the disk and the clamps. No heat flux occurs along the axle supporting the disk. The temperature distribution of the disk was found as outlined in [4]. (Figure 2)

Figure 2 Temperature distribution of disk heated to 100 °C at r_B rotating at 100 rad/s

The disk was simulated for temperatures at r_B from 0°C to 200°C at intervals of 20°C. For each temperature the disk was simulated at rotation speeds varying from 0 to 300 radians per second.

Simulations were then done to determine the effect of thinning the disk. Both the steel and composite disks were simulated at decreasing thicknesses. This analysis was completed with rotation speeds of 100, 200 and 300 radians per second and a temperature of 100 °C at r_B .

For the finite element analysis the disk was modeled using shell elements. For the composite portion shell element capable of simulating laminates were used. First a structural analysis was performed to calculate the stresses due to rotation and thermal expansion (including the shape memory effect). These stresses were used as prestresses for the modal analysis. The block Lanczos method was used for the modal analysis.

5. Results

For the steel disk the stresses in the disk are as found in the following graph. For the unheated disk, the hoop stresses are tensile along the disk. As the temperature at the r_B is increases, the hoop stresses near r_B become compressive.

Figure 3. Hoop stresses of steel disk rotating at 100 rad/s

For the composite disk we see that when the disk is heated, the hoop stresses remain mostly tensile near r_B .

Figure 4. Hoop stresses of composite disk rotating at 100 rad/s

The finite element analysis shows that using the shape memory alloy did result in improvements in the critical speed of the disk. These improvements are comparable to results obtained for pretensioning. The hoop stresses for the composite disk near r_B are kept at 0.

The SMA is able to counter the hoop stresses in the disk near r_B for the temperatures between the phase transformations. As a result the SMA inserts the onset of critical speed due to temperature increase is found to be higher by the phase transformation temperature range ($T_{Af} - T_{As}$).

Figure 5. Comparison of fundamental frequency of disk rotating at 100 rad/s.

Figure 6. Comparison of fundamental frequency of disk rotating at 200 rad/s.

Figure 7. Comparison of fundamental frequency of disk rotating at 300 rad/s.

Table 1. Comparison of disk thickness between steel and composite disk for similar fundamental frequencies.

Angular Velocity (rad/s)	Fundamental Frequency (rad/s)	Composite disk thickness (mm)	Reduction in disk thickness
100	77.7	3.55	29%
200	32.2	3.72	25.6%
300	21.5	4.41	11.8%

The use of SMA inserts in the rotating disk allowed a thinner disk to operate with similar fundamental frequencies as outlined in table 1.

6. Conclusion

The proposed method of intergrating SMA inserts into spinning disk subject to heating along the periphery shows promise. The tensioning performed by the SMA is proportional to the temperature of the disk and in turn to the compressive hoop stresses of the disk. The proposed method would also allow the disk to remain in a near constant state of stress during variable operating condition. Also the temperature of the critical speed of the disk is increased by the transformation temperature range ($T_{Af} - T_{As}$).

7. Nomenclature

r_A = non dimensional radius a, a/b

r_B = non dimensional radius b, b/b

ω_{cr} = Critical speed

ω_n = Frequency of vibration

ω_0 = Speed of rotation (rad/s)

T_{As} = austenite start temperature

T_{Af} = martensite end temperature

7. References

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