Basic Electricity Courses Revisited

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Abstract

Teaching basic electricity courses to engineering students from other fields is a difficult task. Several factors compound the problem such as: large student groups, lack of interest, the high level of abstract thinking required, the varied topics to be covered and the use of complex arithmetics. New approaches have been implemented to alleviate the problem. Amongst them, avoiding the use of complex numbers significantly reduces the student apprehension and passive resistance to the topic.

1 Introduction

The subject of electricity, when it must be introduced to students from other specialities presents a number of challenges, some particular to the subject, some shared with the other subjects also being taught to people from various specialities, therefore having diverging main interests.

The problems that are shared with other subjects have mainly to do with lack of interest, since the students from one speciality naturally fail to see how subjects from a different field can seriously affect their competence in their chosen field.

Problems which are peculiar to electricity stem from the highly abstract nature of the subject and the complex mathematical tools required to approach it.

Finally such cross field courses are often attended by very large populations, and disinterest is an infection that spreads easily in large groups.

Some solution is needed to ensure the students attain the level of expertise required by their particular programme.

2 Re-thinking the programme

Depending on the speciality of origin, students from other fields may have to take electricity courses as soon as their first semester and sometimes as late as their last year. Obviously their levels of preparation may vary widely, which makes for a very mixed crowd. It is difficult to keep the interest of some without risking total alienation of others.

Further, the students tend to consider these courses as secondary and therefore, they tend to modulate their efforts and do just enough to pass, or more precisely, to not fail.

Finally, geography plays an important role, since the professors teaching those courses are generally based in the electrical engineering department, which can be quite remote from the students usual area of activity. This creates a physical separation which is not conducive to positive interactions.

It follows that the program should be adapted to this situation, and that some means of stimulating students' interests should be found. To this end, a number of avenues of solution exist, some of which are:

- Make sure to use problems and examples that connect well with the students' field of interest
- Develop laboratory experiments which are relevant to said fields of interest. for example, experiences in electrolysis for chemical engineering students, analogue simulations of a car suspension for mechanical engineers etc.
- Suggest problems and exercises based on actual data they are likely to encounter in their fields, specifications of real motors, pumps, breakers etc. for example.

3 The powers approach

3.1 Why is electricity special?

First of all, contrary to every other subject encountered so far by those students, we have absolutely no intuitive idea of the variables involved.

While everyone has a feel for forces and an idea of the effects of mass and inertia, which makes Newton's law palatable, everybody has seen springs and their applications to measuring forces, (who has not seen a fish scale?) the notions of current and voltage are foreign to our sensible experience and can only be approached via fairly abstract mathematic tools. Hence the problem. It is difficult enough to grasp the meaning of an equation describing a falling rock, but what happens when you break a loop containing an inductor and a low voltage source is totally counter intuitive. Add the fact that the usual approach to solving electricity problems involves differential equations and complex numbers, two subjects a good proportion of the students may not have heard of yet, and the problem becomes obvious.

3.2 What can be done about it?

First of all, let us see what the average non electrical engineer would want to know about electricity.

For the mechanical engineer and to some extent the industrial engineer, electricity is that thing that makes machines work. It is therefore useful to understand enough about it to choose motors, actuators, etc. for a given application, and to be able to specify suitable supply choices from the many available.

For the chemical engineer and also the materials engineer, electroplating (to protect some metals from corrosion), cathodic protection against aggressive environments and electrolysis to extract and purify some materials are of utmost importance.

Since the above groups constitute the bulk of our, shall we say "customers", I will not list more.

As it turns out, contrary to the needs of electronics or signal processing engineers it is quite possible to approach the majority of problems encountered by these groups without ever (or hardly ever) resorting to differential equations or complex arithmetics, provided one is willing to accept a few notions on faith, i.e. without formal justification. The mathematical tools that have been developed for analyzing electrical circuits should require some justification, but while the electrical engineer should be critical of them, and familiar with the reasons they work and the conditions under which they might not, the casual user is justified to accept them.

3.3 What do we then propose?

We will attempt to approach the problem strictly from the point of view of power, rather than to play around with currents and voltages when at all possible.

With very few exceptions, these students are primarily users of electricity, and their concerns are mainly with the quality, suitability and safety of the supply.

Unfortunately, of late, devices have made an appearance which complicate matters somewhat and make it even more important that the essential concepts be presented as simply as possible, so as not to totally lose these non specialists. We are referring here to new types of loads and sources, converters, distributed energy supplies etc.

Fortunately however, most of these can be approached from the point of view of power without losing much generality.

Depending on the supply available, the machinery or equipment may differ considerably, but a watt is a watt whatever the source that produces it, and that what we eventually pay (or charge) for.

If we accept that what matters is the average power produced or used, it matters little that the supply is direct current, although that has become non existent, single phase alternating current at whatever frequency, or multiphase current, a watt remains a watt and it turns out that most problems can be solved without much more than that fairly simple notion.

For the sake of argument, we shall proceed to compare the classical approach using complex numbers and an approach using only notions of power, applied to the same problem.

3.4 Classical approach.

Classically, supply problems involving ac power are analyzed by using phasors, which are described by complex numbers. These are limited to linear type loads, but this constitutes most real problems. It is therefore given that all voltages and currents are sine waves at the supply frequency and can be fully described by their amplitude (or some function of such) and their phase, relative to some arbitrary time reference.

Unfortunately for our particular crowd of neophytes, these techniques require the constant use of complex numbers, which are, at best, unfamiliar and at worst scary to them, and the significance of which mostly escapes them. It is interesting to note that when students, are asked what they got from the notion of phasors they most often cite that it was their introduction to complex numbers. This is rather backwards, from our point of view, since the course was meant to make use of supposedly familiar complex numbers to solve somewhat difficult electrical problems, not to

use said electrical problems to learn about complex numbers.

To a non electrical engineer, the interest is mainly the magnitude of currents and voltages involved, so the proper cable sizes and insulation levels can be specified. Little does a mechanical engineer care that motors can be modelled by complex numbers, provided one can figure out how much current will be needed, and therefore what cabling should be specified.

The effect of phase differences throughout the circuit will mainly result in power factor (pf) reductions, and this value can usually be got by reading the nameplate of the equipment.

Of course, from our point of view, things would be much simpler if Edison had won his point and everything were direct current, but the practical problems that come with direct current far outweigh the advantages of simpler calculations.

As it is, calculations involving ac current require the introduction of notions of “apparent” power, and the related notions of “real” (or “active”) power and “reactive” power, which some like to call “imaginary”.

At this point, it must be taken on faith that total power in a system, that is the algebraic sum of all powers consumed (conventionally taken as positive) and produced (conventionally taken as negative) is zero.

It must also be taken on faith that this holds true for both real and for reactive power, as imaginary the latter might be.

Figure 1 could represent almost any circuit constituted of an energy source and a load connected by a line. It could be a dc circuit, which we shall not consider, a single phase ac circuit, or a single-line representation of a balanced three phase circuit.

Figure 1

Working with phasors, the load (generally inductive) would be represented by its impedance (Z_L). This impedance is a complex value and may be evaluated from the nominal data for the load, namely its nominal apparent power (S_{Lnom}), nominal voltage (E_{Lnom}) and nominal power factor (pf_{Lnom}).

$$Z_L = \frac{E_{Lnom}^2}{S_{Lnom}} \angle -\cos^{-1}(pf_{nom}) = |Z_L| \angle \phi_L$$

The line connecting the load to the source is represented by a line impedance (Z_l) which is usually inductive and may be evaluated from cabling characteristics and the geometry of the installation. Usually it can, of course, also be measured.

$$Z_l = \sqrt{(R_l^2 + X_l^2)} \angle \tan^{-1}\left(\frac{X_l}{R_l}\right) = |Z_l| \angle \phi_l$$

The power source, always sinusoidal, is considered ideal, therefore unaffected by the load.

All currents and voltages can therefore be represented by phasors which are either known, as is usually the case for source voltage, or to be calculated, as is the case for most other variables in the circuit. These phasors are shown as bold case values in figure 1.

Let us assume that the current problem is to determine what the source voltage should be to assure that the load operate in nominal mode, which is desirable. Different approaches are possible, we shall chose to use what seems to be the simplest in this case, namely the voltage divider.

$$E_L = E_{Lnom} \angle 0^\circ$$

$$E_s = \frac{Z_L + Z_l}{Z_L} E_L$$

$$E_s = E_s \angle \theta_s$$

Source current may be calculated s follows :

$$I_s = I_L = \frac{E_L}{Z_L}$$

$$I_s = \frac{E_{Lnom}}{|Z_L|} \angle -\cos^{-1}(pf_{Lnom})$$

$$I_s = I_s \angle -\phi_L$$

Real and reactive powers at the source and the power factor as seen by the source are calculated as follows:

$$P_s = E_s I_s \cos(\theta_s - (-\phi_L))$$

$$Q_s = E_s I_s \sin(\theta_s - (-\phi_L))$$

$$pf_s = \cos\left(\tan^{-1}\left(\frac{Q_s}{P_s}\right)\right)$$

If power factor correction is desired, a capacitor may be added as shown in figure 2. The same method can then be used to evaluate the required source voltage after this capacitor is added.

Figure 2

The impedance of a capacitor is a function of its capacitance (C) and the operating frequency (f) according to the following expression:

$$Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{1}{2\pi f C} \angle -90^\circ$$

The imaginary operator (j) reflects the fact that the power within a capacitor is entirely reactive (imaginary) and the “minus” sign indicates that the capacitor, by convention produces rather than consuming reactive power.

The source voltage is therefore obtained as follows:

$$Z_{eq} = Z_L // Z_C = \frac{Z_L Z_C}{Z_L + Z_C} = |Z_{eq}| \angle \varphi_{eq}$$

where $//$ is used to indicate parallel impedances

$$E_S = \frac{Z_{eq} + Z_l}{Z_{eq}} E_L$$

$$E_S = E_S \angle \theta_S$$

And the source current is calculated as follows:

$$I_S = \frac{E_L}{Z_{eq}} = \frac{E_{Lnom} \angle 0^\circ}{|Z_{eq}| \angle \varphi_{eq}}$$

$$I_S = \frac{E_{Lnom}}{|Z_{eq}|} \angle -\varphi_{eq} = I_S \angle -\varphi_{eq}$$

Real and reactive powers (P_S and Q_S) as well as the power factor (pf_S) seen by the source are calculated as follows:

$$P_S = E_S I_S \cos(\theta_S - (-\varphi_{eq}))$$

$$Q_S = E_S I_S \sin(\theta_S - (-\varphi_{eq}))$$

$$\text{pf}_S = \cos\left(\tan^{-1}\left(\frac{Q_S}{P_S}\right)\right)$$

Calculations performed on the circuits of figures 1 and 2 are valid for single phase circuits as well as for “Y” connected balanced three phase circuits, since the latter can be considered as being three identical single phase circuits. Delta connected circuits can be shown to have an equivalent “Y” connected alternative, and therefore no loss of generality results from this approach. The voltages appearing in this model are phase voltages, and the currents are line currents. Obviously, if the diagram actually represents a three phase circuit, the powers calculated will be one third of the total circuit power.

3.5 Power based calculations

Techniques based on power (real and imaginary) conservation allow us to calculate the currents and voltages involved without the need to refer to phasors or any complex numbers. However, the notion of reactive power, which is peculiar to electrical engineering, is really based on complex numbers, but once you accept the principle of its existence, you do not have to confront them. This notion will take care of the complexities without actually facing them.

To compare the two approaches, let us apply the power conservation method to the above problem. To that end, let us refer to figure 3, which is the same as figure 1, except for the fact voltages and currents are now represented by scalar values. For practical reasons the value used for each quantity is its rms value rather than its amplitude.

The only impedance which remains in the circuit is the actual line impedance which cannot be avoided in these calculations. This is in fact based on complex numbers, but while they almost rear their ugly heads here, we can actually avoid directly using them by only considering the real (R_l) and imaginary (X_l) parts of the line impedance (Z_l). Those are obtained from the characteristics of the cables, the geometric arrangements and the materials used.

Figure 3

First the real and reactive powers consumed by the load can be obtained from the nameplate.

$$P_L = P_{Lnom} = S_{Lnom} \cdot pf_{Lnom}$$

$$Q_L = Q_{Lnom} = P_{Lnom} \cdot \tan(\cos^{-1}(pf_{Lnom}))$$

or, recalling trigonometry, $Q_L = P_{Lnom} \sqrt{\frac{1 - pf_{Lnom}^2}{pf_{Lnom}^2}}$

The current consumed by the load is, by definition:

$$I_s = I_L = \frac{S_{Lnom}}{E_{Lnom}}$$

The line consumes reactive and real powers according to the relationships below:

$$P_l = R_l I_s^2 \quad \text{and} \quad Q_l = X_l I_s^2$$

Since powers have to be conserved the source has to supply real and reactive powers to both the load and the line:

$$P_s = P_L + P_l, \quad Q_s = Q_L + Q_l, \quad \text{so} \quad |S_s| = \sqrt{P_s^2 + Q_s^2}$$

The source power factor is therefore:

$$pf_s = \cos\left(\tan^{-1}\left(\frac{Q_s}{P_s}\right)\right) \quad \text{or} \quad pf_s = \frac{P_s}{\sqrt{P_s^2 + Q_s^2}}$$

This, knowing the current and apparent power at the source allows us to calculate the required source voltage:

$$E_s = \frac{|S_s|}{I_s} = \frac{\sqrt{P_s^2 + Q_s^2}}{I_s}$$

If the problem is one of power factor compensation, capacitors will have to be added as in figure 2, which becomes figure 4 in the power approach. The added

capacitor, considered ideal, is represented by its nominal reactive power (Q_{Cnom}) at nominal voltage (E_{Cnom}).

Figure 4

The reactive power for the capacitor is:

$$Q_C = Q_{Cnom} \times \frac{E_L^2}{E_{Cnom}^2}$$

And the current from the source :

$$I_s = \frac{\sqrt{P_L^2 + (Q_L - Q_C)^2}}{E_s}$$

Real and reactive powers consumed by the line are:

$$P_l = R_l I_s^2 \quad \text{and} \quad Q_l = X_l I_s^2$$

Real and reactive powers supplied by the source are:

$$P_s = P_L + P_l \quad \text{and} \quad Q_s = Q_L + Q_l - Q_C$$

The power factor seen by the source is:

$$pf_s = \cos\left(\tan^{-1}\left(\frac{Q_s}{P_s}\right)\right) = \frac{P_s}{\sqrt{P_s^2 + Q_s^2}}$$

And the required voltage at the source is :

$$E_s = \frac{|S_s|}{I_s} = \frac{\sqrt{P_s^2 + Q_s^2}}{I_s}$$

Let us consider the problem of a three phase balanced system, as represented in figure 5. The powers are now the total power for all three phases. The currents are line currents and the voltages are line voltages.

Figure 5

These calculations remain valid whatever the type of connections, be they star, triangle or mixte of the load and compensation

Real and reactive powers for the three phase load are, assuming nominal line voltage:

$$P_L = P_{Lnom} = S_{Lnom} \times pf_{Lnom}$$

$$Q_L = Q_{Lnom} = P_{Lnom} \times \tan(\cos^{-1}(pf_{Lnom}))$$

$$\text{or, } Q_L = P_{Lnom} \sqrt{\frac{1 - pf_{Lnom}^2}{pf_{Lnom}^2}}$$

The reactive power produced by the capacitors is: :

$$Q_C = Q_{Cnom} \frac{E_L^2}{E_{Cnom}^2}$$

The line current supplied by the source is:

$$I_s = \frac{\sqrt{P_L^2 + (Q_L - Q_C)^2}}{\sqrt{3} \times E_s}$$

The real and reactive powers absorbed the line are:

$$P_l = 3 \times R_l \times I_s^2 \quad Q_l = 3 \times X_l \times I_s^2$$

The real and reactive powers supplied the source are:

$$P_s = P_L + P_l \quad Q_s = Q_L + Q_l - Q_C$$

And the power factor, as seen by the source is:

$$pf_s = \cos\left(\tan^{-1}\left(\frac{Q_s}{P_s}\right)\right) = \frac{P_s}{\sqrt{P_s^2 + Q_s^2}}$$

The line voltage at the source has to be:

$$E_s = \frac{|S_s|}{\sqrt{3} \times I_s} = \frac{\sqrt{P_s^2 + Q_s^2}}{\sqrt{3} \times I_s}$$

Both approaches yield the exact same results, but the powers method does so without having to deal with complex numbers except, very obliquely, through the inevitable reference to real and imaginary parts of line impedances.

4 The mystical powers of ICTs

Information and Communication Technologies may be used to help alleviate the physical separation between teachers and students by giving them a meeting place in hyperspace.

- Web sites dedicated to the subject matter and updated daily are an excellent meeting place.
- Weekly mandatory home assignments can assure that students keep up with the subjects, even if they have been absent from class. This has the advantage of raising an alarm to the student who is falling behind while it is still possible to catch up.
- With hundreds of students, it is all but impossible to give speedy feedback about their state of learning. To alleviate this we are experimenting with automated evaluation of homework assignments.
- Other resources may be offered such as collections of solved and unsolved problems, practice problem generators, photographs, videos and texts related to the subject matter and their specialities.

5 Conclusion

The reluctance demonstrated by students from other fields for basic electrical courses may be significantly reduced by combining different technological tools with the elimination of complex arithmetics. This seems to constitute a promising approach to positively modify the student perception of the subject. Websites, forums, interactive exercises and online homework assignments with rapid feedback constitute powerful tools if correctly and moderately used together with a good textbook. Replacing complex arithmetics by an approach based on power flow rather than complex impedances shows promise.