A DECISION-MAKING FRAMEWORK FOR ENGINEERING MATHEMATICS EDUCATION: A CRITICAL LITERATURE REVIEW

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Abstract – *Engineering is a fascinating blend of mathematics, science, and creativity, which helps to solve some of the world’s leading problems. This paper asserts that mathematics is central to engineering because it is the primary means by which engineers are able to model systems, as well as understand the consequences of their intended actions, designs, and operations. Thus, it follows that educators inherit an ethical responsibility to elucidate the importance of mathematics as a central theme to interventions in the world in general and to engineering in particular. This paper provides a strong theoretical overview of the existing and potential innovations that can be made to traditional undergraduate engineering mathematics instructions and further discusses the perceived gaps in the literature. A framework is proposed for teaching engineering mathematics: mathematics is the primary means by which ethical, responsible decisions can, and should, be made and it is thus crucially important that it be appreciated and taught as such within engineering programs.*

Keywords: Mathematics, Engineering, Decision-Making

1. INTRODUCTION

All engineers have a background in mathematics; it is impossible to receive a professional engineering license without having pursued post-secondary mathematics courses. Thus, it is important to consider why mathematics is so important for engineers, how it should be taught to engineering students, and whether the importance of mathematics is being adequately communicated. What follows is a preliminary framing exercise that examines such teaching from perspectives including educational theory, motivation, coaching, and ethics. Placing engineering education within such a framework is an important step before moving on to the evaluation of specific teaching methodologies, which will allow for a more robust determination of how engineering students can best learn to use mathematics effectively and powerfully in their careers.

How and when engineers employ mathematical approaches is clearly and inextricably rooted both in engineering education and practice. Many of the mathematical skills and methods that are useful in engineering professions are taught in undergraduate programs, from the ‘oft-dreaded’ first-year calculus sequence to later courses in such topics as numerical methods, differential equations, and probability. Perhaps more importantly, though, it is through undergraduate education that future engineers develop an understanding of the applicability of such skills and methods in order to formally anticipate consequences without having to first experiment and/or actually implement designs in potentially dangerous ways. Thus, it is crucial that the teaching of mathematics stress and elucidate its importance to engineers, rather than simply teach mechanical proficiency. Since the early 1900’s the discussion of mathematics as it relates to engineering has been at the heart of discussion; ‘mathematics is the very bone and sinew of an engineering course’ [35, p.501]. Still true today, mathematics is the key tool that humans have at their disposal for making these predictions, and thus, for making decisions that are documented, vetted and defensible, taking into account both positive outcomes and potential negative consequences, and thus making decisions that can truly be characterized as ethical and responsible.

2. LITERATURE REVIEW

The following literature review covers shifts in mathematics education in the past half century, the emerging and increasingly prevalent themes of metacognition, curriculum structure, and motivation, and recent initiatives to address mathematics education within engineering programs.

2.1 Shifts in Mathematics Education

There have been major shifts in mathematics education in North America since the mid-twentieth century, particularly affected by the cognitive revolution and the growth in understanding of how people learn. Schoenfeld

[32] outlines the history of the multiple shifts in American mathematics curricula and approaches, which have bounced back and forth between a traditionalist back-to-basics focus on teaching facts and knowledge, and a reformist focus on problem solving and process. Perhaps fuelled by the concern over international rankings of mathematics proficiency in recent years, there have been repeated calls for an examination of, and changes in, mathematics education; the debate predictably pits traditionalists, who call for the practice of basic skills, against reformists, who call for discovery learning in the classroom to help students construct their own knowledge. While these wars show no particular signs of nearing resolution, many [e.g., 3, 8, 32] argue for a middle ground between teaching tried-and-true basic mathematics skills in the classroom and allowing students to explore, discover, and innovate.

While these debates are strongly focused on K-12 education, writings on the teaching of post-secondary mathematics also occur. Indeed, many authors and educators are taking a more holistic view of students and of teaching, as educational efforts are being examined from a variety of perspectives. For example, Merriam, Caffarella and Baumgartner [27] examine how and where adults learn, addressing both more formal educational settings (such as universities and other post-secondary classrooms) and less formal ones (such as personal internet research). They outline theories of learning and cognitive development that apply to adult education. Of note is their discussion of Piaget's four stages of cognitive development (which culminate in formal or abstract thought) and a suggested fifth stage of post-formal thought (in which adults reflect on formal thought and its subjectivity). This is important to consider in any conversation about post-secondary mathematics education, particularly because undergraduate students will not necessarily all have reached this stage; while some students recognize the tentative and uncertain nature of, for example, a mathematical model, others consider knowledge to be more absolute; engineering educators can account for this in their classrooms to help students gain perspective and understanding.

Further, Bransford, Bowen and Cocking's [5] text on *How people learn* – as well as associated texts, such as *How students learn* by Donovan and Bransford [12] – stress that it is important that educators consider their courses from four perspectives (termed student-centered, knowledge-centered, assessment-centered, and community-centered) in order to develop comprehensive and effective lessons that take into account the needs of students and of course curricula, as well as the benefits and limitations of evaluating work and learning in collaboration with others. Again, this is an important point of discussion in post-secondary mathematics

education, where the academic focus is often skewed towards curriculum content and student assessment.

Finally, there has been significant discussion of engineering mathematics education from studies by the International Commission on Mathematical Instruction, whose mandate covers mathematical teaching at all levels. Of particular relevance is a publication on modeling and mathematical applications [4], which is a collection of essays and reflections on the applications of mathematics in the real world. The authors stress such points as:

- The importance of integrating formal instruction with hands-on experiences rather than treating them as separate curricular components [1]
- The value of exposing students to the experimental and exploratory aspects of mathematics [30]
- The idea that modeling is not simply a demonstration of the usefulness of mathematics but rather can be presented as a way in which mathematics supports enterprises in other areas [10]
- The importance of teaching the process of modeling and not only importing models into the classroom [26, 33]

These ideas apply current theories of student learning to education and are all pertinent to post-secondary mathematics education, especially within engineering programs where modelling is the primary mathematical endeavor.

2.2 Metacognition

A common thread of metacognition runs through much of the literature pertaining to engineering mathematics education. Indeed, this thread also runs through several recent and more popular books. A number of publications in the past few years encourage readers to better understand their own thought processes, often in order to avoid common pitfalls and make better decisions. For example, Lehrer's *How We Decide* [25] looks at decision-making as two complementary processes of reason and emotion, and discusses how to understand and promote one's brain activity in order to make better decisions. In a similar vein, Kahneman's *Thinking, Fast and Slow* [19] describes mental heuristics and strategies that the brain often uses (for better or for worse), and explains how to take an outsider's perspective of one's own thinking in order to take advantage of these strategies when they are beneficial and avoid them when they are not. More recently, Ellenberg's *How Not to Be Wrong* [14] suggests that mathematical thinking is simply the extension of common sense, which allows us a lens through which to examine the world and a means by which to make informed decisions, if we can understand how to use it effectively. The development of metacognitive strategies is a topical discussion, and no less so in engineering programs than in popular culture and in elementary and

secondary education; this discussion suggests an ethical imperative to teach mathematics in a meaningful way in engineering education.

In examining critical thinking skills, Kuhn [23] suggests that “to achieve [...] control of their own thinking is arguably the most important way in which people both individually and collectively take control of their lives” (p. 23). From a perspective of engineering education, what is perhaps just as important is how engineers individually and collectively take control over their environment. Highly developed metacognition can allow engineering students to understand how they learn, to enhance and deepen their learning, to justify what they know, to understand what (and the extent to which) they don’t know, and to compare and evaluate various assertions and approaches. Metacognition is highlighted in academic literature as a way in which students can develop expertise in a domain. Indeed, Bruer [7] suggests that novices, as they work to become experts, can be “intelligent” novices by employing metacognitive skills; the act of thinking about one’s thinking and learning leads to greater awareness of the learning process and thus better learning. Merriam, Caffarella and Baumgartner [27] similarly suggest that students can work to become experts through metacognition, reflecting on what their minds are doing as they learn and solve problems. In the same vein, Ericsson [15] proposes that students become experts through deliberate practice, which includes the metacognitive activities of setting specific goals, designing/monitoring learning activities, and reflecting. Finally, Jonassen [18] explains that metacognition involves both knowledge of cognitive processes and the regulation of them, and reports that students with higher metacognitive awareness are better problem solvers, regardless of aptitude, suggesting that these skills can compensate for lower aptitude. Such metacognitive skills are of prime importance in undergraduate engineering programs, and no less so in mathematics courses in which students must develop logical, ideally expert, approaches to questions and problems.

Educators have a responsibility to help students develop these metacognitive strategies and become better learners by taking control of their own learning. While stating that students can be more effective learners if they use metacognition, Donovan and Bransford [12] also stress that students rarely engage metacognitive strategies. Kuhn [24] elaborates on this idea by suggesting that students are typically not aware of how they incorporate new material, nor are they in control of the process; they do not consciously interpret new evidence and evaluate whether their conclusions are based on the new evidence or on prior beliefs. Teachers must bridge this gap by teaching students to reflect on their own thinking and learning. For example, in mathematics, procedures are often taught in ways that encourage correctness rather

than connectedness. When learning mathematics, students can employ metacognitive strategies to assess their answers, but only if they have learned to think using number sense [12]. Kuhn [24] asserts that metacognitive control is something that develops with age and over time, and educators must provide a context in which this development can emerge so that students can gain control of their thinking. In particular, mathematics should be taught to engineering students in such a way that they develop an awareness of their own knowledge and thinking; through this, they will become better able to learn in meaningful ways and to make well-informed, justifiable, ethically responsible decisions.

2.3 Curriculum Structure

Course content, usually stated as a list of course objectives, is often considered, especially by students, as a long list of topics to be covered; students would often struggle to give a one or two sentence summary of a particular course that focuses on only the guiding principles of the course. One would hope that the teachers of such courses would be able to do so, but that may not be unfailingly true. Bransford, Bowen, and Cocking [5] and Donovan and Bransford [12] emphasize that it is important to teach students not just subject content, but a framework with which to understand the discipline. Factual knowledge needs to be situated in a conceptual structure in order to be properly understood. Unfortunately, Donovan and Bransford [12] also suggest that the skills and processes that students learn are rarely placed within a framework of core organizing ideas; teachers must make this structure explicit in order to help students situate their learning contextually. Effective teachers need to have not only content knowledge, but also the pedagogical knowledge of how to structure and teach their subject area.

Of course, each discipline has its own big ideas, its own organizing framework and a unique structure. Within pure mathematics, for example, internal structure and logic, and consistency, are centrally important. However, within engineering, ethically responsible decision-making based on sound physical and mathematical models is an important organizing idea. Moreover, Bruer [7] asserts that domain-specific expertise that does not readily transfer to other domains. Thus, in order to be prepared for a career in engineering, students need to be taught within the context of the discipline; mathematics as taught within one discipline must differ from mathematics as taught within another. Mathematics as taught to engineers must move beyond computational proficiency and focus on core organizational principles, such as ethical and responsible decision-making.

Teaching within a framework of a handful of guiding principles can strengthen students’ abilities to return to

first principles when working through problems. It is remarkably powerful, particularly in mathematical problems, to be able to work through a problem based on its fundamental structure rather than jumping straight to a prepared, procedural method. While it can indeed be more time-effective to use shortcuts, the ability to recall guiding principles and use them can ground solutions and subsequent decisions in the reasons why the methods are applicable. Thus, an educator's ability and efforts to structure engineering mathematics courses within a clear framework is an important theme throughout the literature; we argue that that structure should include ethical and responsible decision-making.

2.4 Motivation

Motivation is another important theme pertinent to undergraduate engineering programs, as motivation is essential for persevering under challenging circumstances. Engineering students require academic motivation when facing complex projects and courses throughout their programs; practicing engineers require similar motivation when working on intellectually challenging projects, as well as more subtle motivation to persist in lifelong learning when external factors, such as grades, are no longer present. While many undergraduate engineering students may begin their programs motivated and willing to work hard in their courses, challenges to this motivation – such as dense, technically difficult mathematics courses – inevitably occur. In order to succeed it is beneficial for students to, (a) believe that their intelligence is flexible and can in fact grow, and (b) be interested in growing their intelligence in the area in question.

On the first point, Dweck [13] reports that students are more likely to persevere at challenging tasks when they perceive their own intelligence as flexible, rather than unchangeable. Indeed, there has been a shift away from perceiving intelligence as a single, static, quantifiable trait towards perceiving it as a set of behaviors and habits of mind that are fluid throughout one's lifetime [11]. In order to prepare students for careers in engineering, educators must help students recognize the importance of lifelong learning and perseverance.

On the second point, Hidi and Harackiewicz [17] report that interested learners pay more attention to the subject matter, persist under difficulty, and enjoy learning more than non-interested learners. They go on to distinguish between individual interest and situational interest: individual interest is what we typically think of when we consider a person who is internally interested in a topic, and so more inclined to learn about it; situational interest, on the other hand, is generated by elements in the external environment. While educators cannot control the level of individual interest of students, they do have some

control over generating situational interest by how they present material; studies have shown that situational interest can be enhanced by, for example, presenting material in a more connected, meaningful context and affording students choices in their learning. Further, situational interest (triggered by, for example, an interesting and meaningful lecture) can lead to individual interest. Thus, whether or not students come into a mathematics course with individual interest in the material, educators can help them develop and sustain such interest by emphasizing the connections to and role within engineering.

Once students recognize the fluid nature of their own intelligence and are interested in growing that intelligence in a particular area, they can set (whether intentionally or not) learning goals for themselves. In literature on motivation, learning (or mastery) goals are distinguished from performance goals: with learning goals, people strive to deepen their understanding of a topic, whereas with performance goals, they strive to perform well at a task. Dweck [13] shows that students have tended to show more perseverance and risk-taking in response to challenging tasks when they are in pursuit of learning goals over performance goals. Not surprisingly then, performance goals and external motivators have often been vilified in literature in favor of learning goals and internal motivation; however, a more nuanced approach suggests that they can help students focus their attention and develop interest in a particular subject or area, and thus can actually precede learning goals [17].

Our current educational system is designed in many ways to foster the setting of performance goals over learning goals: there is great competition to gain acceptance in universities in general, and competitive programs such as engineering in particular, as well as later importance placed on grade point averages and class placement in order to gain recognition, scholarships, and/or coveted supervisors or projects. These performance (grade) goals are extrinsic motivators, and are in some respects unique to education. In sport, for example, champions may believe they can be champions but that is seldom their goal. Athletes are motivated by intrinsic goals, 'to be the best they can be,' to push limits and boundaries [6, 31, 34]. Dweck [13] states that better learning occurs when there are intrinsic goals. Blending intrinsic goals into mathematics education within engineering could be as simple as relating mathematics within engineering; students would have a desire to be better at mathematics because it would make their engineering courses potentially easier to follow along in.

External motivators such as grades are similarly much less prevalent in the workplace, especially when it comes to tasks such as learning to apply mathematical methods. Thus, it is important that students gain experience in setting learning goals as well as performance goals; the

literature [13, 17] suggests that this can be achieved by developing students' understanding of their own fluid intelligence, and by helping them generate individual interest in the areas in question. Within engineering programs, this can be better achieved if mathematics is taught in a more meaningful engineering context by instructors who can help students to recognize growth in their intelligence and abilities.

2.5 Integrated Engineering Programs

There have been some recent initiatives to address concerns in mathematics education within engineering programs. So far, the program with the most significant impact across multiple institutions is the Wright State model. The Wright State model, piloted in 2004 and described originally in the paper *Rethinking Engineering Mathematics Education: A Model for Increased Retention, Motivation and Success in Engineering* [21], replaces the traditional first-year calculus course with an Engineering Mathematics course. The successes of the model are well-documented in terms of student retention and motivation, with a recent longitudinal study reporting that since the inception of the model graduation rates have increased, graduation GPAs have improved slightly, and the engineering curriculum has become more accessible, especially to underrepresented groups [22]. The model has expanded throughout the United States, with a team representing 17 institutions piloting similar programs as of 2013.

Ultimately, programs like this move away from the perspective that engineering students should be proficient in mathematics prior to their engineering education, and address mathematics instead as a useful tool within the program. The engineering context provides students with a tangible reason to learn the mathematics, placing it within the framework of the discipline. With the success of these programs, the movement to make the connection between mathematics education and engineering seems to be making real headway.

3. PERCEIVED GAPS IN THE LITERATURE

The above literature review identifies and expounds areas that are pertinent to engineering education; while not exhaustive, these areas were identified during both formal and informal meetings between faculty and students at a Canadian university interested in further exploring and improving engineering education. It became clear during these discussions that, much has been written about the theories and research around mathematics curricula, metacognition, motivation, and coaching, and these areas were further explored using online database searches, reading lists from education courses, and current news reports. However, it became

clear both through the engaging discussions and the literature review that there are still areas that have not been addressed in the literature. In particular, there is little if any discussion of the fundamental reasons for which mathematics is important to engineering; in other words, there is little that has been written that addresses the unique power of mathematics to inform responsible skill execution, to ground engineering in ethical considerations, and to make engineering more accessible to students who are not typically drawn to the field, such as by blending coaching principles in with engineering and/or mathematics engineering education.

3.1 Distinguishing Mathematics and Engineering

It is useful to briefly discuss what mathematics is, and specifically how it is both distinct from but related to engineering. One way to distinguish broadly between the two fields is to emphasize the fact that mathematical endeavors tend to move toward abstraction while engineering endeavors move towards the applied or concrete (see Fig. 1). Mathematics seeks to create general, symbolic, abstract representations of entities and relationships. While these may indeed start in the real world – that is, people recognize patterns and connections around them – they do not end there; mathematics moves towards describing them in an abstract way. On the other hand, engineering takes generalities and applies these to particular projects. Engineers use the rigorous, artificial worlds created within mathematics to change the physical world we live in.

Fig. 1. While mathematical endeavors move “upwards” from the concrete to the abstract, engineering endeavors move the opposite way.

An example can help to elucidate this distinction. The Finite Element Method, a mathematical method fundamental to areas such as mechanical engineering, was developed in the mid-twentieth century in order to address structural engineering problems. The method, which finds

approximations to differential equations, was rigorously formalized by the 1970s. Importantly, this formalization allowed the method to be used confidently in other applications; in particular, the method is applicable not just in structural analysis but is generalizable to all fluids, and thus can be used on projects relating not just to, for example, bridges and dams, but also to spacecraft and aircraft. Thus, the Finite Element Method is used in many branches of engineering, such as civil, mechanical, and even chemical or biomedical engineering.

Figure 2 situates the Finite Element Method between the concrete world where the specific applications are made and the abstract world where the general methods are formalized. The method is further positioned as a specific instance of weak solutions theory, which allows for the reformulation of differential equations in order to find approximate solutions. Another specific instance of weak solutions is also shown: The Finite Volume Technique, which leads to weak solutions for which quantities such as mass and energy are conserved. This technique is applicable to such diverse engineering areas as fluid mechanics and biological processes.

Fig. 2. The Finite Element Method and Finite Volume Technique are both particular methods encompassed by Weak Solutions Theory, and which both have many engineering applications. Mathematical rigor allows engineers to move throughout the network of theories and techniques to find new and innovative solutions to complex problems.

Note that it is precisely such mathematical rigor which allows engineers to move along the branches of the network of theories and methods. Thus, the ability to generalize a theorem – to make the structure of the problem abstract – is what allows that structure to be reused in other areas. This allows engineers to approach complex problems in innovative ways, making decisions that are confidently grounded in rigor. Thus, engineers need to learn to recognize that highly abstract thinking can turn out to be deeply and profoundly useful in practice. Understanding the role of mathematics in decision-making is actually one of the best arguments for insisting on mathematical rigor in an engineering curriculum.

3.2 Exercising Skills Responsibly

Engineering graduates finish their degrees with a huge amount of skill and technical expertise. Importantly, engineers are expected to apply these skills thoughtfully rather than blindly; that is, they cannot simply follow a set of instructions to guide them through skill application, but must carefully consider how to apply their skills to each unique project on which they work. Mathematical thinking is really the only way to do this.

Mathematics can of course be applied less thoughtfully; practicing engineers could apply their mathematical formulae to each project without critical consideration of why the mathematics works and is applicable. However, rigorous mathematics allows engineers to understand each project deeply. That is, it allows practitioners to see beyond the details of the project to its structure, to the essence of how it works. Without this understanding, engineers risk misapplying a formula – using it in a situation to which it is not applicable, perhaps because the timeframe is too long, the geography is unsuitable, the formula does not account for certain conditions, or maybe even overcompensates for them – and making a poorer decision than they might have.

On the other hand, the true power of mathematics is that it allows users to go back to first principles, to confirm why a particular method applies, or what part of a method applies, and to make predictions and decisions that are tailored to each individual project. Thoughtful mathematics leading to the responsible application of skill helps to bridge the divide between our emotional and rational brains. Too often, mathematics courses are taught as though mathematics is an entirely rational process; crunching numbers allows us to override our emotions, to formally weigh the options, and to make a decision based on numbers and facts. However, as Lehrer [25] points out, people are incapable of making decisions without engaging their emotions as well. In order for students to engage their emotional minds more actively in decision-making and in mathematics, it is important that they develop a sense of estimation and approximation. Estimation – the ability to think about a problem and have an underlying sense of the magnitude and approximate value of the quantities involved – is extremely powerful for determining a reasonable range of answers to a question; it helps students to notice when a calculation error has been made as well as to determine whether their data is within the limits of a particular model. Developing students' skills of estimation, though not highlighted in much literature, should be done concurrently with formal and rigorous mathematics in order to give students the tools they need to apply their skills and make decisions responsibly in future engineering careers.

Further, mathematics allows engineers to exercise their skills more responsibly under uncertainty. An engineer's job requires making informed predictions about future situations – whether accounting for potential future weather, urban growth, or technological use. Thus, engineers by necessity deal with uncertainties. Instead of relying on gut instincts and guesses, engineers benefit by putting reasonable bounds on their estimates and communicating these to the team with whom they are working, so that there is no illusion that projections are absolute. Exercising one's skills responsibly entails not only offering informed decisions and recommendations, but also recognizing the limitations of these.

While it may be common knowledge that engineers must make responsible decisions and use their skills effectively, it appears that few have explicitly identified mathematics as the means by which to do this. In fact, it is only through mathematics that engineers can understand the fundamental structure of a problem, make sound ethical projections about the future, and understand the limitations and bounds of these.

3.3 Grounding Engineering in Ethics

About ten years ago, the National Academy of Engineering published two documents [28, 29] that predicted the roles of engineers and engineering in society in 2020, given that the world and technology are changing rapidly, and made recommendations for the profession and for academic faculties of engineering. In particular, these documents emphasize the growing need for engineers to make ethical decisions that take a broad view of the world we live in; they conclude that “it would be particularly helpful if the engineering community could successfully communicate the social context of engineering – how engineering has made enormous contributions to our quality of life – and the social responsibilities of engineers beyond just taking care to exercise their skills responsibly” [28, p. 27]. Both documents highlight the importance of future engineers setting high ethical standards.

Ethical decision-making suggests that engineers must work not only to ensure that they are accountable to governing bodies, but also to ensure that they explore the consequences of possible alternative decisions, advice, and actions, and how these could affect people and the environment over long periods of time. Catalano [9] points out that few codes of ethics for professional engineers in the United States address what it means to act ethically regarding the societal challenges of security, poverty, and the environment; at most, codes suggest vaguely that engineers should protect the public and the environment. However, ethical decision-making nevertheless is crucial to the engineering profession. To act ethically and responsibly as an engineer is not only to

be truthful and thorough in one's work; it is also to anticipate how one's work may engender change in people's lives and in the environment, and thus to make decisions and provide counsel that lead to positive changes.

The discussion of how mathematical reasoning can address this concern is lacking. However, a mathematical approach is most important when making decisions that have societal repercussions, decisions that affect not just a few people but many. Karney and Mather [20] make this point in addressing how engineering mathematics can inform public policy decisions:

But what precise role does mathematics play in such cases? It is at least this: when collective action is desired or likely to be rewarded or punished, there are huge potential benefits or incentives to formally and consciously anticipate both human and natural responses prior to our proposed actions. Such a “formal method of anticipation” has a more common name, and it is simply modeling. Our models act as stand-ins or substitutes for the reality of direct action and, in the 21st century, such models are almost invariably hybrid mathematical and conceptual constructs showing through symbols an association between actions (decision variables) and consequences (output or design variables). [p. 47]

Mathematical modeling allows engineers to anticipate consequences, and thus to promote positive outcomes, avoid catastrophic ones, and weigh various options against each other. This is precisely the basis of ethical decision-making. Without formally anticipating how a project is likely to affect the community and the environment, no ethical decision can be made. Mathematics allows for a robust approach to the evaluation of alternatives, and thus leads to responsible decision-making grounded in ethics.

3.4 Making Engineering More Accessible

Mathematics courses are often a significant hurdle for many students pursuing engineering degrees. Gains in primary mathematics achievement have resulted from student engagement in afterschool math-based activities, including math-stories and -apps [2]. However, by the time students are independently learning in an undergraduate engineering program they far less likely to engage in mathematical activities outside of the classroom to deepen their mathematical understanding, partially due to the time constraints engineering students face.

Indeed, first-year calculus has been identified as a major barrier to students' continuation in engineering programs [16] and part of the reason that many students drop out of – or perhaps, though evidence is more

difficult to find, decide not to enroll in – engineering programs. This, however, is very likely to be influenced by how mathematics is framed and motivated within these programs. Currently, mathematics appears to be treated as either (a) an end product in and of itself, or (b) a useful tool for engineers.

The former perspective can be characterized as detrimental to prospective engineers. In this perspective, the point of learning mathematics is to master the methods and pass the course. This can occur when mathematics courses are divorced from engineering and taught in a way that isolates them from physical applications. If the point of a mathematics course is presented as simply needing to master the methods, students are being set up to either gain misplaced confidence in their abilities or to become demoralized and even leave the program. In other words, mathematical proficiency should not necessarily be equated with engineering proficiency; being good at one does not necessarily translate into being good at the other.

The second perspective is generally a more positive one. Mathematics is presented to students as something that they can draw on in their professional practice. If they need to, say, determine the size of a pipe or troubleshoot an electrical circuit, they should be able to access an appropriate formulae or approach. Indeed, with mathematics, the answer is often considered to be absolutely accurate and precise, one which is “superior” to an estimate based on experience or informed guesses; engineers can be more confident in their answers, and therefore their decisions and counsel, if they have applied the appropriate mathematical methods. However, while this perspective seems to view mathematics as an important part of engineering, it fails to recognize any deeper reasons that mathematics is used.

A more useful framing than either of these is one that highlights mathematics as a means by which ethical and responsible decisions can be made. Currently, engineering is stereotyped as a technically-oriented profession characterized by conclusions that are more likely to be black and white than nuanced by societal perceptions and communication with policy-makers. However, a more student-centered perspective that frames curriculum around central ideas could help: if faculties were to structure their programs so that engineering was highlighted as a profession that interacts meaningfully with society through ethical project design, and if mathematics was highlighted as the fundamental way to evaluate the consequences of alternative decisions in these projects, this could help mathematics to be perceived less as a hindrance and more as a lens through which engineering relates to the world.

This third perspective on mathematics – that it is the primary way by which engineers explore the beneficial and detrimental consequences of alternative options in

order to make important decisions that have societal repercussions – might make engineering more accessible to those traditionally not attracted to the field. That is, such a framing of mathematics might attract those who aren’t naturally drawn to the abstract thinking of pure mathematical manipulation or of scientific laws. Such an approach would surely benefit from accessing both formal and informal approaches to thought and problems solving, seeking both confirmation and discrepancy while engaging with the world.

4. CONCLUSION

We are at a critical point in engineering education; graduates of engineering programs need to be prepared to make ethical and responsible decisions regarding how they work with and manipulate the environments and materials around them, as these decisions can and do affect large numbers of people, often over wide geographic areas and long time frames. Mathematics is the primary means by which such decisions can and should be made; it is thus crucially important that it be appreciated and taught as such within engineering programs.

As major decision makers in a rapidly changing world, and in a world where decisions must increasingly consider issues of sustainability and future impacts, it is critical that engineers make informed, defensible, responsible, and ethical decisions. Mathematical models and approaches are the primary means by which such decisions are made. It is important that engineering students learn to recognize this and to value and use mathematics in this way, rather than viewing mathematics courses as simply providing useful tools that will come in handy in the future. Teaching mathematics within an appropriate framework can make this explicit to students and help them develop the metacognitive skills and motivation to apply mathematical methods effectively, and thus better prepare them to be more ethically responsible engineers.

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