MEASURING THE CONNECTION BETWEEN MATHEMATICS AND ENGINEERING

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Abstract – *Mathematics forms the foundation for all the engineering disciplines. Students have trouble transferring this mathematical knowledge from their mathematics classes to the rest of their undergraduate engineering classes. This study is borne out of a desire to 'be better,' to endeavour always to try to improve, but first, you need to know where one the starting point. The authors are also passionate about mathematics as it relates to engineering. Anecdotally the authors had heard that both students and faculty were disappointed and aggravated with the current status of mathematics teaching in undergraduate engineering. With no known study in Canada looking at how mathematics connects with engineering the authors went down the path to find out how strong the connection between mathematics and undergraduate engineering is at the University of Toronto.*

Through a mixed-method survey, the goal was to measure respondents' (i.e. The teaching staff) views on the importance of and students' competence of both mathematical topics and specific mathematic skills. A survey was administered in the 2017 fall semester to all of those who teach in the Faculty of Applied Science at the University of Toronto. The first part of the survey used a 5-point scale, the second part of the survey had open-ended questions.

The responses to the 5-point scale questions demonstrate that the selected mathematic topics and specific skills were all seen as important and that the students' competence was lower than their rated importance. The open ended-questions asked for respondents definitions and views as they related to abstract, applied, and engineering mathematics.

Keywords: Mathematics, Engineering Mathematics, Curriculum

1. INTRODUCTION

Mathematics forms the foundation for all the engineering disciplines; it is the first of the graduate

attributes from the Canadian Engineering Accreditation Board, a division of Engineers Canada [1]. The connection between mathematics and engineering that is the ability for students to transfer their knowledge between mathematics courses and other engineering courses, has been documented as difficult for students [2, 3, 4, 5, 6]. Both sides of the teaching equation blame the other for this transfer challenge: some engineering faculty blame the math faculty for not teaching effectively (linking mathematics teaching to engineering); conversely, mathematics faculty often say that the students themselves do not even try to learn mathematics [6]. We do not believe that either of these is accurate but are curious to see if a partial truth might be hidden here.

The goal of our study was first and foremost to see how the Faculty of Applied Science at the University of Toronto (FASE) might do better through a lens of mathematics as it relates to undergraduate engineering. This study is borne out of an existing study from the Ohio State University. Colleagues in the Department of Engineering Education wanted to measure the connection between engineering and mathematics. We developed a survey similar to that used at Ohio State with a few minor modifications [6]. Specifically, we updated the departments so that they reflected those in the University of Toronto's Faculty of Applied Science and Engineering. Modifications were also made to the first question of the survey to include a question related to broad mathematical skills, which were not initially included in the Ohio State work, but which their own survey suggested were relevant.

The adapted survey was issued in late 2017 to engineering faculty and staff at the University of Toronto. The intended outcomes (the research questions) of the survey sought to answer two specific questions:

- 1) Which mathematical concepts are regarded as the most important for success by faculty?
- 2) How successfully do faculty believe their undergraduate programs are at building student competence in the same area (as above in Question 1)

The results of the survey are currently under review, as are the interviews that were borne out from select faculty who wished to participate in interviews.

This study is a mixed methods study; the survey that was sent to faculty in the 2017 fall semester was a blend of both qualitative and quantitative measures. The survey was six questions; the first two asked what department the teaching staff was affiliated with, the second question asked for their teaching title (i.e. Professor, Associate Professor, Assistant Professor, or Sessional Lecturer). The next two questions were a matrix-style question, on a 5-point scale to ask about the importance of and students' competence of the mathematical topics and specific mathematical skills. Finally, there were four open-ended questions asking about the teaching staffs' views on terms related to engineering mathematics, including, abstract mathematics, applied mathematics, engineering mathematics, and finally space where respondents could share any other thoughts.

A mixed method study was chosen to tell the narrative of what was occurring in FASE before we suggested any recommendations or suggestions.

2. THE PROTOCOL

The protocol was administered through the Dean's office in the 2017 fall semester. Initially, the intention was to leave the survey open for a response over a three week period, the same period of time as The Ohio State University, but we decided to leave the survey open for a longer period to allow more faculty to respond [6]. The initial email to participate in the survey was sent on November 7, 2017, with a follow-up email on December 8, 2017. The survey was open for just over six weeks; the survey closed on December 22, 2017, the final day of the fall semester. The survey was administered using an online tool (i.e. Survey Monkey) and as an attachment in the email that the Dean sent out; there were five individuals who chose to email back a copy of the survey the rest were collected online.

2.1. The Sample Population

All of the approximately 261 teaching staff (Professors, Assistant Professors, Associate Professors, and Sessional Staff) in the eight listed departments were invited to participate in the survey [7]. The survey was meant to be anonymous; no specific information was collected (i.e. Name, email address, address, phone number, office number, etc.). Two pieces of information were collected, how they identified as part of the teaching staff and department affiliation.

2.2. The Importance of and Students' Competence

The survey sought out to the following questions:

- 1) Which FASE department are you associate with?

- 2) What is your job title?
- 3) Which mathematical concepts are regarded as the most important for success by faculty?
- 4) How successfully do faculty believe their undergraduate programs are at building student competence in the same area? (as above in Question 1)

These two questions were asked twice, once for a set of mathematical topics, and, secondly, for a set of specific mathematical skills.

In the survey the questions were formulated as follows:

- Question 1 - How important is it for students who graduate from the University of Toronto undergraduate engineering program to be able to competently apply mathematics concepts from each of these areas listed?
- Question 2 – How competent (i.e. what is your perceived competence) is the average graduate from the University of Toronto undergraduate engineering program in the following areas?

2.3. Open-Ended Questions

Finally, there were open-ended questions regarding the teaching staff's views on terms as they related to mathematics in undergraduate engineering. Space was provided to provide the teaching staffs' views on the following topics:

- Abstract Mathematics,
- Applied Mathematics, and
- Engineering Mathematics.

At the end, we had space where we allowed respondents to offer any other thoughts, suggestions, or comments as they relate to mathematics as it is taught in undergraduate engineering in FASE.

3. RESULTS AND DISCUSSION

3.1. The Sample Population

The response rate of the survey of the approximately surveyed teaching staff was 63 (24%). The survey was meant to be anonymous; no specific information was collected (i.e. Name, email address, address, phone number, office number, etc.). Two pieces of information were collected, how they identified as part of the teaching staff and department affiliation.

The Importance of and Students' Competence and **Error! Reference source not found.** show the distribution of respondents. One thing we did not account for was emerging departments that were not listed on the FASE website when the survey went out; The Department of Engineering Communications Program (ECP) is a recently recognized department in FASE, both of the 'Other' respondents below in Table 1 are from ECP.

Table 1: Representation of Percentage of Respondents in FASE Completing the Survey; Representation of the percentage that each Department makes up in FASE

Department	Responses	FASE
ChemE	22.22%	11.11%
CivE	15.87%	16.09%
MSE	1.59%	6.13%
MIE	20.63%	21.84%
EngSci	1.59%	3.45%
ECE	30.16%	29.12%
UTIAS	4.76%	6.90%
IBBME	0.00%	5.36%
Other	3.17%	0.00%

The above titles stand for (in order as listed): Chemical Engineer (ChemE); Civil and Mining Engineering (CivE); Materials Sciences and Engineering (MSE); Mechanical and Industrial Engineering (MIE); Engineering Sciences (EngSci); The Edward S Rogers Department of Electrical and Computer Engineering (ECE); The University of Toronto Institute for Aerospace Studies (UTIAS); and Institute of Biomaterials and Biomedical Engineering (IBBME).

At the time of comparing the representation of respondents and their respective departments, the ECP program was not listed, as mentioned above. As such the numbers for *Other* likely do not correlate to today's representation of departments in FASE.

Most notably Chemical Engineering (ChemE) is over-represented in this study, and The Department of Material Sciences (MSE) is under-represented as compared with the makeup of FASE.

Table 2: Representation of Percentage of Respondents' Job Titles; Representation of Percentage of Job Titles that make up FASE

Job Title	Respondents	FASE
Professor	55.56%	55.17%
Associate Professor	30.16%	20.69%
Assistant Professor	11.11%	13.79%
Sessional Lecturer	0.00%	10.34%
Professor Emeritus	3.17%	0.00%

Two of the respondents to the survey identified themselves as *Professor Emeritus* under the heading of *Other*. There is an over-representation of respondents from *Associate Professor* and an under-representation of *Sessional Lecturers*. One thought the authors had around sessional lecturers was that they might not have been included in the Dean's mailing list, and hence, may not have known about the survey.

3.2. Importance of Mathematical Topics and Perceived Competence of Graduates

There were 11 mathematical topics included in the survey. Respondents were asked to respond on a 5-point scale. An option of 'Not Applicable' was made available as it related to student competence; the thought was that if the topic was unimportant then the respondent would likely not comment on a students' competence. The results are shown below in Table 3.

Table 3: Summary of Faculty Ratings of Importance of and Students' Competence with these 11 Mathematical Topics. Ratings are on a 5-point scale. Numbers in parentheses are standard deviations.

Topic	Average Importance	Average Competence
Calculus	4.20 (1.08)	3.20 (0.96)
Linear algebra	3.82 (1.18)	2.59 (1.04)
Analysis of nonlinear phenomena	3.10 (1.08)	1.69 (1.04)
Ordinary differential equations	3.75 (1.27)	2.59 (1.18)
Partial differential equations	3.23 (1.27)	1.92 (1.15)
Complex variables/functions	2.97 (1.25)	1.78 (1.11)
Numerical simulations	3.74 (1.26)	2.33 (1.12)
The use of vectors to represent physical quantities	3.70 (1.26)	2.46 (1.17)
Discrete Math	3.02 (1.20)	1.65 (1.13)
Numerical Solutions	3.61 (1.14)	2.44 (1.05)
Statistics	3.70 (1.16)	2.20 (1.02)

One of the limitations of the study is noted above; there was an assumption that if the topic was unimportant that a respondent would not comment on students' competence. Also, there were limitations on assessing students' competence; bias would play a role here. Assessments and marks were not chosen to be used in this analysis; there is too much subjectivity with an assessment from year-to-year and test-to-test.

More analysis will be done at the granular level, specifically how the various departments rated the importance of and students' competence of these mathematical topics in the future.

Calculus was rated as the most important, but student competence was only rated as average; in the theme of 'being better,' student competence could improve. Other topics including Linear Algebra, Ordinary Differential

Equations, Numerical Simulations, and Statistics were also rated as important by faculty. Following in the same fashion as Calculus, the perceived competence of the students was lower than the rated importance. These results suggest that faculty do not believe that students are mathematically prepared in the mathematical topics listed, which important for an undergraduate engineering degree.

3.3. Importance of Specific Mathematical Skills and Perceived Competence of Graduates

There were 12 specific mathematical skills included in the survey. Respondents were asked again to respond on a 5-point scale. An option of 'Not Applicable' was made available as it related to student competence; the thought was that if the topic was unimportant then the respondent would likely not comment on a students' competence. A summary of the Specific Mathematical Skills data is shown below in

Table 4: Summary of Faculty Ratings of Importance of and Students' Competence with these 12 Specific Mathematic Skills. Ratings are on a 5-point scale. Numbers in parentheses are standard deviations.

Table 4: Summary of Faculty Ratings of Importance of and Students' Competence with these 12 Specific Mathematic Skills. Ratings are on a 5-point scale. Numbers in parentheses are standard deviations.

Specific Mathematical Skills	Average Importance	Average Competence
Evaluating solutions/checking work	4.49 (0.80)	3.30 (0.79)
Being familiar with units and dimensions	4.56 (0.84)	3.42 (0.95)
Knowing how to create and interpret graphs	4.51 (0.82)	3.12 (1.00)
Performing algebraic manipulations	4.05 (1.14)	3.12 (0.98)
Knowing how to convey & interpret engineering relationships through mathematical expressions	4.32 (0.99)	2.61 (0.98)
Using parameters/symbols, rather than numerical values, in analysis	4.27 (0.93)	3.12 (1.05)
Interpreting the role of parameters in mathematical expressions	4.17 (0.99)	2.72 (1.00)
Formulating mathematical models	4.14 (1.02)	2.26 (1.04)
Being familiar with estimation/knowledge of magnitude scales	4.39 (0.81)	2.84 (0.99)
Knowing when to differentiate or integrate	4.12 (1.18)	2.75 (1.06)

Working with multivariable problems	3.98 (1.04)	2.25 (1.22)
Creating algorithms	3.64 (0.98)	2.18 (1.04)

The same limitations applied in this part of the study as in the mathematical topics section (section 3.2).

All 12 specific mathematical skills were rated as important by the faculty respondents. Similar to the above the ratings on students' competence in these skills were lower, as assessed by the respondents. The same granularity will be applied as well, as mentioned above in section 3.1.

3.4. The Open-Ended Questions

A few targeted open-ended questions were asked of the respondents after the questions on the importance of and the students' competence; not surprisingly respondent rates were lower in these open-ended questions (38 of the 63 [60%] respondents answered these). These questions asked about definitions as they related to mathematics in undergraduate engineering. We asked for definitions on:

3.4.1. Mathematics

Before we get into the responses and their definitions it is useful to first generally describe mathematics. Generally speaking, mathematics is "the science of numbers and their operations, interrelations, combinations, generalizations, and abstractions; and of space configurations and their structure, measurement, transformations, and generalizations; Algebra, arithmetic, calculus, geometry, and trigonometry are branches of *mathematics*" [8]. Summarized one might say, as seventy-five prestigious Canadian and American mathematicians of 1962 eloquently said, "to know mathematics means to be able to do mathematics: to use mathematical language with some fluency, to do problems, to criticize arguments, to find proofs, and, what may be the most important activity, to recognize a mathematical concept in, or to extract it from, a given concrete situation" [9].

3.4.2. Abstract Mathematics

The responses were interesting, there was a divergence amongst responses; the respondents either described what applied mathematics was or what it was not. The respondents agreed that abstract or pure mathematics (as it was phrased in the survey) had to do with 'theory,' 'proofs,' 'generalization,' and 'logic.' It was the starting point to solve applied mathematics problems. Some of the respondents went on to say that it is the mathematics "without specific application" (the antithesis of applied mathematics).

In general, mathematics seeks to create symbolic, abstract representations of entities and relationships [10]. It is the notion of abstract mathematics, the theory, the pureness of it that allow us to create models (applied mathematics) to represent the world. Abstraction is "this

kind of notion of taking the essence of a real-world objects... It is ideas and concepts, and the general idea behind the abstraction or abstracting something is that you're taking it away from the particular concrete real world and you're going more into the direction of ideas and concepts" [11].

3.4.3. Applied Mathematics

Applied mathematics was agreed upon by the respondents that this is the mathematics in the 'real-world.' It was called the mathematics that 'solves problems,' problems rooted in engineering and sciences.

Applied mathematics is the sibling to abstract mathematics. "Applied Mathematics involves using mathematics to solve real-world problems, in particular, problems in scientific research and the development of new technology. Applied Mathematics also involves the development of new mathematical and computational tools" [12]. These are mathematical structures representing some part or aspect of the physical world by providing representations through similar models [13]. Traditionally applied mathematics has been affiliated with the mathematics used in the professional domains including but not limited to economics, engineering, physics and statistics [14].

3.4.4. Engineering Mathematics

Engineering mathematics had a bit of split in how the respondents described this 'field' of mathematics. Some of the respondents (10) agreed that engineering mathematics was a subset of applied mathematics. The remaining respondents agreed it was the mathematics that solved engineering problems.

"Mathematics is the very bone and sinew of an engineering course" [15]. "Mathematics is an important component in the field of engineering. Thus, Mathematics education at the school level should lay the groundwork for effective study and application at the professional ranks" [16]. To this point mathematics is often taught in the first years of engineering, students are expected to complete courses in both calculus and linear algebra [7, 17, 4].

3.4.5. Common Themes Amongst All the Questions

A theme emerged throughout all the open-ended questions, that mathematics, as it is taught to engineers, should be taught less as 'a means to an end' and more as a tool, specifically a communication tool. Engineers are asked to solve some of the world's most complicated problems. It's not just about the solution, it's the ability of an engineer to communicate their solution, which allows that solution to become a built reality; if you cannot convince someone that your solution is effective, efficient, safe, etc., through communication your solution will not get built. As we zoom away from an engineering career back to the undergraduate engineering classroom, we should ask this, *what if we taught mathematics to*

undergraduate engineering students as a way of communicating?

4. CONCLUSION

Engineering mathematics forms the foundation for student success; thus, students must have the ability to do mathematical modelling as part of engineering design to create sustainable and ethical solutions to the challenging problems our future societies may face. Engineering mathematics is the language that is needed to convincingly illustrate these technical solutions. The key question here is whether this message is being effectively communicated to engineering students. Perhaps instead of teaching mathematics as an aspect of the undergraduate engineering curriculum and instead we taught mathematics as a communication tool, through mathematical modelling, students' competence levels would increase to match the level of importance as rated by the teaching staff (i.e. the respondents).

In the future, the goal is to administer this survey, or something similar, to teaching staff at other Canadian Institutions. Another goal is to seek out an intervention to help students see not just the value in mathematics, but to see it through a different lens, as a method to communicate their designs not just through written word but also through mathematics.

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